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Toward a scientific and personal biography of Tullio Levi-Civita (1873–1941)

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Abstract

Tullio Levi-Civita was one of the most important Italian mathematicians of the first part of the 20th century, contributing significantly to a number of research fields in mathematics and physics. In addition, he was involved in the social and political life of his time and suffered severe political and racial persecution during the period of Fascism. He tried repeatedly and in several cases successfully to help colleagues and students who were victims of anti-Semitism in Italy and Germany. His scientific and private life is well documented in the letters and documents contained in his *Archive*. The authors' aim is to illustrate the events of his life by means of his large and remarkable correspondence.

Sommario

Tullio Levi-Civita fu uno dei più importanti matematici italiani della prima parte del ventesimo secolo e contribuì in modo significativo a numerose discipline in campo matematico e fisico. Inoltre, egli partecipò alla vita sociale e politica del suo tempo e fu vittima delle severe persecuzioni politiche e razziali del periodo fascista. Egli tentò in più occasioni, e talvolta con successo, di aiutare colleghi e studenti che erano vittime dell'anti-semitismo sia in Italia sia in Germania. La sua vita scientifica e personale è ben documentata nelle lettere e nei manoscritti contenuti nel suo *Archivio*. Lo scopo degli autori è di illustrare i fatti salienti della sua vita facendo uso della sua ampia e notevole corrispondenza.

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Introduction

In the following we present a sketch of the life and work of Levi-Civita based largely on the letters and manuscripts that are kept in the Archives of the *Accademia dei Lincei* in Rome.[1] These documents—partially published in [Nastasi and Tazzioli, 1999, 2000, 2003]—represent one of the largest testimonies on Italian mathematics in the first part of the 20th century. The letters and reports in Appendixes help to illustrate some important aspects of Levi-Civita's life and work that are too long to be reported in the text of the paper (see [Galletto, 1973; Hodge, 1942]).

We consider both Levi-Civita's fundamental scientific contributions and some important events in his personal life. In fact, he developed his principal researches concerning the theory of relativity, the three-body problem, adiabatic invariants, hydrodynamics, and tensor calculus during the First World War. While Nazism struck German mathematics from the thirties onward, Fascism in Italy during the same period obliged professors to swear fidelity to the government (1931) and the Racial Laws that were promulgated (1938). From 1938 till his death, Levi-Civita as a private person helped colleagues and friends under persecution and sometimes succeeded in finding new positions for them in South America or in the United States. In his correspondence there are traces of all these historical events together with his research in different mathematical fields.

Levi-Civita's professional biography is divided into two great periods—the period at the University of Padua, where he graduated in 1892 and began his extraordinary scientific career, and the period at the University of Rome, where he arrived from Padua in 1918 and created a *school* of mathematics at an international level. The substantial difference between the two parts of our paper reflects this dichotomy. In particular, the first part is mainly devoted to Levi-Civita's scientific contributions, while the second part describes his Roman years, the international appreciation of his work that developed, and his political and social role.

1. Levi-Civita at the University of Padua

Levi-Civita studied at the University of Padua, where he was a student of Giuseppe Veronese (1854–1917) and Gregorio Ricci-Curbastro (1853–1925); he graduated in 1892 under the direction of Ricci-Curbastro with a dissertation on the theory of invariants. During his years in Padua, Levi-Civita concluded his scientific education and embarked on research on a number of topics that were to become fruitful research fields for him and several students of his at the University of Rome.

1.1. The three-body problem

The classic three-body problem was one of the principal research fields in Levi-Civita's career; a good survey of his works on the subject is contained in [Barrow-Green, 1997]. Levi-Civita was interested in the three-body problem for over 20 years and derived a regularization of the differential equations in the neighborhood of a binary shock by starting from some results of Paul Painlevé (1863–1933). The first of several papers by Levi-Civita on this topic was published in 1903 in two notes in the *Comptes Rendus*

[1] In the Archives there are about 5000 letters sent by a thousand mathematicians throughout the entire world.

[1903a, 1903b]; in a subsequent paper [1903c] he put together the results of these notes and characterized the singular trajectories in the restricted problem.

Some years later Levi-Civita [1906] was able to remove the singularities and rationalized his results from [1903c]. He also found that if the bodies in question are real celestial bodies—and not treated as material points—then the motion remains regular only if there is no collision and the distances between the bodies are not below a certain given limit. By regularizing his equations, Levi-Civita obtained an analytic representation of all possible arcs of a trajectory inside a *sufficiently* small region D around P (P is the point where the system is regularized). But the minimum distance between D and P could not be found.

Karl F. Sundman (1873–1949) finally achieved a complete solution of the three-body problem [1907, 1909, 1912] with a very simple method. Though his 1912 paper in *Acta Mathematica* was highly appreciated by his contemporaries, Sundman did not consistently maintain his interest in the topic. Nevertheless his ideas influenced many papers, the most notable of which was a simplification due to Levi-Civita [1920] on a canonical regularization of the three-body problem in the neighborhood of a binary collision.

It was the director of *Acta Mathematica*, Gösta Mittag-Leffler (1846–1927), who asked Levi-Civita to write the paper, a fact revealed in the correspondence between the two men.[2] On November 11, 1916, Mittag-Leffler wrote to Levi-Civita:

> Cher et très honoré Collègue,
>
> Votre article dans les Comptes Rendus du 25 avril de cette année “Sur la régularisation du problème des trois corps” [Levi-Civita, 1916] me paraît extrêmement remarquable. Je vous propose de m’écrire un article sur ce sujet pour être publié dans les Acta Mathematica. Votre mémoire si intéressant qui a ouvert la voie pour toutes ces recherches a été publié dans les Acta [Levi-Civita, 1906] ainsi que la lettre de Weierstrass à moi du 2.2.1889 publié dans le tome 35 des Acta [Mittag-Leffler, 1912] et l’article de M. Sundman [Sundman, 1912]. Il serait donc, je trouve, très bien si j’aurais l’occasion de publier encore vos nouvelle recherches.

It is also possible to follow some fundamental steps in the history of the three-body problem in the letters between George David Birkhoff (1884–1944) and Levi-Civita from 1913 to 1935.[3] In 1912 Birkhoff opened a new era in the history of dynamical systems by proving Poincaré’s well-known “last geometric theorem.” In his last paper devoted to the three-body problem, Poincaré [1912] had announced a theorem which, if shown to be true, would confirm the existence of an infinite number of periodic solutions for the restricted three-body problem. Shortly after Poincaré’s death Birkhoff [1913] was able to prove his theorem in a very elegant way. In this connection, Levi-Civita wrote to Birkhoff on January 31, 1913:

> Monsieur et très honoré Collègue,
>
> Je vous remercie vivement pour le bien aimable envoi de vos intéressants mémoires. Je viens de lire avec le plus vif plaisir votre démonstration autant simple et spontanée qu’ingénieuse du remarquable théorème de Poincaré. Veuillez agréer mes félicitations sincères et l’expression de la plus haute estime
>
> T. Levi-Civita

[2] Levi-Civita’s letters to Mittag-Leffler are kept in the *Mittag-Leffler Institute*, Djursholm, Stockholm. The correspondence between Levi-Civita and Mittag-Leffler is published in [Nastasi and Tazzioli, 2000, 323–344].

[3] Levi-Civita’s letters to Birkhoff are kept in *Papers of G.D. Birkhoff*, Harvard University Archives. Courtesy of Andrea B. Goldstein, Reference Archivist. Birkhoff’s letters to Levi-Civita are published in [Nastasi and Tazzioli, 2000, 199–220].

And about one year later, Levi-Civita wrote about Birkhoff's research (letter on January 10, 1914):

> Mon cher Collègue,
>
> Je vous remercie bien cordialement pour votre très aimable lettre et pour le renseignement sur les recherches que vous venez d'accomplir dans le domaine des solutions périodiques. Je sais assez, entre autre d'après ma propre expérience, que bien de chercheurs se sont sentis impuissants devant les difficultés du problème, lorsqu'on tente à l'aborder de face, sans *l'ubi consistam* d'une solution de départ et d'un paramètre à faire varier *assez peu*.
>
> D'autant plus je me réjouis d'apprendre que votre heureuse pénétration vous a permis de surmonter les barrières susdites pour le problème restreint. J'avais déjà remarqué l'annonce de la communication qui s'y rapporte dans le programme du dernier meeting de l'American Math. Society, et j'en attendais le sommaire avec intérêt. Votre lettre a justement prévenu ce désire.
>
> De presque toutes mes publications des années, c'est à peine s'il me reste encore un exemplaire, parfois pas même un seul. Il m'est pourtant bien [...] de vous faire hommage de tout ce que je puis ramener portant (plus ou moins directement) à la mécanique céleste.
>
> Agréez l'expression de mes sentiments dévoués
>
> T. Levi-Civita

Birkhoff's letters to Levi-Civita are generally about Poincaré's last theorem and other scientific questions—such as dynamical systems and the three-body problem—and matters concerning the First World War. About the war, Birkhoff wrote to Levi-Civita on March 7, 1917:

> You refer in your letter to my sympathy for the Allies. I would be ashamed of my country if I did not believe that sympathy of the very deepest kind for their cause is felt by almost all Americans. The vote of Congress the other day is a testimony of this fact. Of all my colleagues at Harvard only two not of pure German blood and German born incline toward the other side; and even these keep very quite. (I might say that I am entirely of Dutch descent, all of my great grand parents being born in the Netherlands.) Unless President Wilson rashly misunderstands American sentiment he will proceed at once to arm our ships and take other necessary steps to uphold our rights upon the seas which the Central Powers have so flagrantly violated. Personally I favor even more rigorous participation on our part. The Germans are a great people of course, but their success would be the defeat of civilization and the best interests of mankind.

Birkhoff repeatedly expressed his sympathy toward Italy and the Allies, writing in various letters that "More generally I hope that America will prove in every way a strong support to the cause of the Allies" (August 10, 1917); "My heart has gone out in sympathy with Italy in her heroic attempt to stem the tide of invasion" (November 22, 1917); "The recent wonderfully thrilling and cheering news of the complete defeat of the attempted Austrian offensive in Italy seemed to me to mark the true turning point of the world war" (July 25, 1918).

Along with his views about the war, Birkhoff described his work in progress and new mathematical ideas. In 1918 he was engaged in fundamental research on dynamical systems and wrote to Levi-Civita (May 1, 1918) from Cambridge (Massachusetts, USA):

> At present I am hard at work with the paper which forms the natural conclusion to the earlier one on Dynamical Systems with Inv[ariant] Degrees of Freedom [Birkhoff, 1920], and have apparently obtained some results which I have been *altogether astonished at*. In particular I believe that I will be able to demonstrate that the periodic orbits of the simplest type in the restricted problem of three bodies are stable in the true sense that, if a certain number is irrational, nearby orbits remain nearby, whereas, if it is rational, nearby orbits remain within a fixed neighborhood of the periodic orbit, although this neighborhood is not "infinitesimal." I have not yet verified all of the analytic details involved in this particular application of the general methods of my paper, which I expect to finish in a couple of months. The result as I have stated it is in harmony with your own fundamental results on instability, but nevertheless it has gone contrary to my preconceived ideas.

On many occasions Birkhoff expressed his profound esteem for the scientific work of Levi-Civita. For instance, he wrote on December 29, 1913 that "your own beautiful treatment of the singular points in

the equations of motion in the restricted problem of three bodies" is "a model of elegance" for his own research. In particular, Birkhoff was very interested in the great compendium on the three-body problem that Levi-Civita would publish in *Acta Mathematica* [Levi-Civita, 1920]. In this connection, Birkhoff wrote to Levi-Civita on July 25, 1918:

> It will be a big help to me if I can make use of your forthcoming paper in the *Acta Mathematica* in preparing my address before the A.A.A.S. It was most friendly of you to suggest sending the proofs as soon as possible. I shall read your paper with the greatest interest.

In the 1930s, at the end of his scientific career, Levi-Civita was interested in the n-body problem from a relativistic point of view. His work on the topic has not been much studied by historians; as an exception we note the paper by Lichnerowicz [1975]. In the twenties, Einstein, Wilhelm de Sitter (1872–1934) [1916], Johannes von Droste (b. 1886) [1926], and Karl Schwarzschild (1843–1921) [1916] had already obtained some important results in the case of the relativistic two-body problem (motion of an infinitely small mass in the field of a given spherical mass). As Marcel Brillouin (1854–1948) noticed (in [Lichnerowicz, 1975, 132]), the main difficulty was to integrate the nonlinear field equations, which were produced by the field interacting with itself. In fact, it is not possible to simplify the equations of motion by introducing a relativistic principle playing the same role as the Newtonian action and reaction principle in classical mechanics. Therefore, it is necessary to develop some general approximating methods in order to deduce the solutions with the given approximation.

Levi-Civita [1937a] published his fundamental paper "The relativistic problem of several bodies" in the *American Journal of Mathematics* of 1937; substantially, it was the text of his lecture given at the *Harvard Tercentenary Conference of Arts and Sciences* on September 4, 1936, which also contains a remark of Einstein (dated November 2, 1936) in Appendix. As Lichnerowicz [1975, 133] wrote, "dans cet article était obtenu de manière cohérente, pour la première fois, le système différentiel régissant les mouvements des centres de gravité de n corps et prenant en compte les parties principales des corrections relativistes."

During the same year and in the same journal, Levi-Civita [1937b] published the text of his other lecture delivered at Harvard University, "Astronomical consequences of the relativistic two-body problem," where he applied his method to the two-body problem and obtained remarkable results about the so-called secular acceleration.

In 1938, Einstein, Leopold Infeld (1898–1968), and Banesh Hoffmann (1906–1986) formulated and solved the same problem using another approximating method [Einstein et al., 1938]. This paper also contains a short note by the astrophysicist Howard Percy Robertson (1903–1961) (in [Einstein et al., 1938, 101–104]). Here Robertson applied the method of Einstein, Hoffmann, and Infeld to the two-body problem and deduced some results on secular acceleration that are different from those already given by Levi-Civita.

It was a mistake in his calculations that led Levi-Civita to the wrong conclusion. This mistake was already noticed by Sir Arthur Stanley Eddington (1882–1944), who in a letter (Cambridge, March 21, 1938) wrote to Levi-Civita:

> Dear Professor Levi-Civita
>
> I am sending you a copy of a paper, which I am proposing to publish in the Proceedings of the Royal Society, which treats the problem of the secular acceleration. As you know I was very interested in the problem. I have an exceptionally able student G.L. Clark, and some of the most vital parts of the paper, especially the discovery of de Sitter's error, are due to him.

You will see that, contrary to your results, we find no secular acceleration; the various terms cancel out as shown in (8.4) of our paper [Clark and Eddington, 1938]. We think there must have been a numerical slip in your calculation which prevented the cancelling (see our comparison with your results after our equation (8.2)).

I realise that it is not possible to arrive quickly at a definite judgement on an investigation; in which so much depends on accuracy in a very long algebraic calculation; but I should be very glad to hear from you, if you have time to examine it. In any case please call my attention to any point on which I may have misunderstood you, or unintentionally misrepresented you; so that I may remedy it in proof.

Whatever the result, it is an interesting problem, which deserves the fullest examination—and incidentally it is associated with very pleasant memories of our time at Harvard.

I have this morning had a visit from Prof. de Mayolo of Peru who was speaking about your visit to S. America.

With kindest regards (...)

Levi-Civita wrote in his answer (March 24) that he needed more time to check his calculations, and was indeed working on a general survey of the relativistic problem of several bodies:

Dear Professor Sir A. Eddington,

I am sincerely grateful to you for your friendly letter of March 21, and for your very obliging attention to communicate to me a copy of the paper (in collaboration with your distinguished student, Mr G.L. Clark) you are about to publish in the Proc. of the Royal Society.

As you obviously think, I am enormously interested in this research and in the fundamental discrepancy concerning secular acceleration, which you find. I shall carefully examine all the matter, in order to detect the origin of the difference of our conclusions. Unfortunately I may never exclude some material mistake in my calculations, though I remember well that I have revised the whole investigation twice, at the interval of one year.

Just in the next days I intended to resume it in order to prepare a detailed exposition to be printed as a little volume in the collection of Professor Villat "Mémorial des Sciences Mathématiques" [Levi-Civita, 1950].

Of course your paper is a strong stimulus to accelerate this work instituting first at all the comparison between my successive steps and yours. (...)

In another letter (in April 1938[4]) Levi-Civita informed Robertson of the paper of Eddington and Clark. Robertson wrote to Levi-Civita on August 2, 1938:

Dear Professor Levi-Civita:

I have now seen the article by Professor Eddington to which you referred in your last card. I find that, on correcting several more or less obvious mistakes, the equations on which his computation is based are the same as those which I obtained on reducing those set up by Einstein, Infeld and Hoffmann. The mistakes to which I refer are the subject of the enclosed copy of a recent letter to Eddington. (...)

[Princeton], June 28, 1938.

My dear Professor Eddington:

I was much interested in the note by yourself and Clark on the 2-body problem, particularly since it substantiates some conclusions of my own. This agreement is all the more satisfying since my work was based on the equations published recently by Einstein, Infeld and Hoffmann, whereas yours is based on a modification of de Sitter's procedure.

Your reduced equations (7.2) are in fact identical with the equations on which my work is based, provided one changes the sign of the coefficient $\frac{4m_1^2m_2}{m^2}$ in the penultimate group of terms in your equation (7.2), and similarly in your equation (7.3)—as seems to be required by your own equation (7.1). [Two typographical errors notes in (7.3) are (1) m_1 in the second term should be replaced by m_2, and (2) the coefficient in question should read $+\frac{4m_1m_2^2}{m^2}$ instead of $-\frac{4m_1m_2^2}{m^2}$.] It happens that this group of terms drops out of the computation concerning the centre of mass of the system—but it affects the equation of the relative orbit, giving rise to a periastron motion $1-16\frac{m_1m_2}{3m^2}$ times that obtained by Levi-Civita and myself! (...)

[4] We do not have such a letter by Levi-Civita to Robertson, but it is mentioned by Robertson himself in his answer to Levi-Civita on April 26, 1938.

In the meantime, Levi-Civita wrote another paper, "Secular acceleration of the mass-center in the relativistic problem of two bodies," where all the calculations were explicitly developed in order that someone could finally show him the mistake in his calculations. The paper was supposed to be published in the volume of the *American Journal of Mathematics* devoted to the centennial of the birth of George William Hill (1838–1914). In the preface of his paper, which was sent to Robertson on October 4, 1938, one reads:

> I have re-examined my previous investigation concerning the secular acceleration of the center of gravity in the relativistic two-body problem, and I have reached the same numerical result as before.
>
> On the contrary, Professor Robertson, applying a quite different method due to Einstein, and independently, Sir Arthur Eddington and G.L. Clark, starting from the same differential equation as I, got the conclusion that (the adopted approximation being in all cases the same) this secular acceleration identically vanishes.
>
> I was obviously strongly impressed by such a discrepancy, which has significant tokens in my disfavour. I bring however myself to publish the details of my work, as I have now written down all intermediate passages, beginning from equations identical with those used by Eddington and Clark (which are fundamentally De Sitter's, duly emended). It will be so possible to control the computations and therefore, in Eddington and Clark's words, "either to detect an error or to obtain further light on the nature of the phenomenon."

Levi-Civita's paper has never been published, since the celebrating volume of Hill's centennial was already printed when Robertson received Levi-Civita's note. However, Robertson and Clark were both in Princeton at this time and determined to check all the calculations in Levi-Civita's paper, as Levi-Civita had requested. Finally, they could show where the mistake was. Robertson wrote in the conclusion of his letter to Levi-Civita on October 20, 1938:

> In view of these results I have thought it best to return your manuscript for your consideration of the points involved. My own opinion is that the procedure of Eddington–Clark (corresponding to that on which my own was based) is the more plausible. I have also discussed this matter with Mr. Clark, who is now in Princeton, and he informs me that he and Eddington stated their *a posteriori* identification of P with Ph so explicitly because they believed that it was essential to perform all differentiations first.

The meaning of the last sentence is well explained in a letter from Clark to Levi-Civita on October 21, 1938:

> Dear Prof. Levi-Civita,
>
> As you will see from the address written above, I am spending a few months in Princeton. I should like to thank you for the copy of your calculations of the two body problem, which has been forwarded from Cambridge.
>
> Professor Robertson and myself were greatly interested in your paper and it occurred to us independently that the discrepancy arose from the term $2\frac{m_i}{r_i}\sum\frac{m_j}{P_{ij}}$ in the g_{44}. In calculating the equations of motion we have put $x_i = x_{\mu i}$ *after* differentiation. If however we put $x_i = x_{\mu i}$ *before* differentiation in the above term (but not in mr_i etc.) we obtain your result.
>
> Accordingly Prof. Robertson has examined your paper with this distinction in mind. He has found that you have indeed followed the alternative course and the difference in our results is accordingly accounted for. I have seen the letter which he is sending to you and fully agree with his comments. (...)

In his answer Levi-Civita recognized his mistake; nevertheless, he trusted in his method and judged it better than Einstein's method. In fact, he wrote to Robertson on November 12, 1938:

> Dear Prof. Robertson,
>
> I am deeply obliged to you for the kind supervision of my paper, which has permitted to you to detect my slip and to re-establish, as a consequence, the agreement with your previous result.
>
> My procedure, though being quite usual, is perhaps worthy of publication, because it furnishes in a few pages all details of calculation. Therefore I dare to send back to you a manuscript, duly emended and retouched. As you will see, at the end of the introduction, I have anticipated the statement that you will have the kindness to add some formulas and remarks, privately communicated to me: I obviously allude to your last letter. With this hope, I beg you to let forward the paper, after your additions and

corrections of any kind, to the Editor of the American Journal, if you do not think, however, that now the thing is quite useless. Of course, I entirely trust to you, thanking you very much for your precious help.

The paper was not published. In Robertson's opinion, "the thing was quite useless." The (correct) method of Levi-Civita was published only in 1950, when the volume of the *Mémorial* was finally printed [Levi-Civita, 1950]. Nevertheless, as Lichnerowicz [1975] pointed out, Levi-Civita was right in thinking that his method was "worthy of publication"; it was indeed more fruitful and natural than Robertson's method, as Vladimir Fock (1898–1974) showed in his works published from 1939 onward, containing the final version of the new celestial relativistic mechanics. After the Second World War the method developed by Levi-Civita and re-elaborated by Fock was extended and became the one usually used in questions of celestial mechanics.

1.2. Hydrodynamics

During his time at the University of Padua, Levi-Civita began studying some hydrodynamical problems, which he and his students further investigated at the University of Rome.

In 1901 he published the fundamental "Note on the resistance of fluids" [Levi-Civita, 1901], where the so-called "hypothesis of wake" is introduced. Such a hypothesis puts on a solid base the law formulated by Newton, according to which the resistance is proportional to the square of the velocity in an incompressible fluid. The hypothesis of wake assumes that a body in motion drags behind it an indefinite liquid column moving with it. Therefore, there are two different regions in the fluid—the wake and the region outside of it—which are divided by a surface of discontinuity. In contrast, if one assumes—as was done *before* Levi-Civita's paper [1901]—that in a perfect fluid a body produces a continuous motion, then the resistance on the body due to the fluid will be *zero* for any shape of the body. Such a theoretical consequence was known as the "paradox of d'Alembert." Thanks to his hypothesis of wake—which according to Henri Villat (1879–1972) is "la seule interprétation acceptable des phénomènes naturels" [Villat, 1918, 47]—Levi-Civita solved the paradox of d'Alembert and deduced an expression for the resistance in agreement with physical experience.

The influence of Levi-Civita's 1901 note is apparent in some letters of Jacques Hadamard (1865–1963), who wrote to Levi-Civita on April 1902:

Cher Monsieur,

J'ai reçu avec grand plaisir votre travail d'Electricité l'Influenza d'un schermo, etc… et vous en remercie vivement [Levi-Civita, 1902]. Il m'est arrivé au moment où j'allais vous écrire relativement à une autre de vos notes, reçue en 1901 et qui m'a tant intéressé, celle qui est intitulée Sulla resistenza dei mezzi fluidi [Levi-Civita, 1901].

Je suis en effet en train de rédiger mes cours professés en 1898–1899 et 1899–1900, lesquels ont précisément pour base la nécessité (absolument générale, et non point relative au seul paradoxe de d'Alembert) de faire intervenir les discontinuités dans le mouvement des gaz. A cette occasion, je compte, si j'en ai le loisir, reprendre l'étude des problèmes sur lesquels vous appelez l'attention dans votre travail. Seulement, il est un cas dans lequel je ne puis être d'accord avec vous : c'est celui des liquides. La véritable théorie du phénomène ne me parait pas pouvoir être cherchée (pour les liquides) dans le discontinuités de l'espèce que vous introduisez. Car ces discontinuités devraient se propager, affectes, à des moments différentes, des molécules différentes (à moins que votre surface $\delta n'$ ait la forme d'un cylindre à génératrices parallèles au mouvement) ce qui est impossible dans le cas des liquides.

Au contraire, dans le cas des gaz, je crois avec vous qu'il y a lieu d'introduire les discontinuités. Sur ce cas de gaz, je vous serais bien reconnaissant si vous pouviez me fournir quelques indications bibliographiques. Le cas des liquides est traité un peu partout, mais sur celui des gaz il doit y avoir d'autres travaux que les très rares que je connais. Pourriez vous, dans ce cas, me les indiquer ? Vous me rendriez grand service.

Recevez, cher Monsieur, l'assurance de mes sentiments bien dévoués

J. Hadamard

And Hadamard added in an undated letter:

> Cher monsieur
>
> Vous avez parfaitement raison : la critique faite à votre hypothèse sur l'existence d'une discontinuité dans le cas des liquides est une simple inadvertance de ma part. Il n'y a, semble-t-il, aucune raison qu'un mouvement discontinu de la nature de ceux que vous considérez ne soit pas possible.
>
> Tout au plus serais-je tenté de ne pas la considérer (en le supposant une fois obtenu) comme résolvant entièrement la question, parce qu'il resterait à savoir comment il naîtrait et surtout—difficulté qui n'existe pas pour les gaz—pourquoi le mouvement continu, qui est compatible avec toutes les conditions du problème ne se produit pas en réalité. A partir d'une certaine valeur de la vitesse, on comprend très bien qu'il en est ainsi. Mais jusque là, je veux dire pour les vitesses suffisamment petites ?
>
> Il est vrai que, pour celles-ci, la chose se présente peut être conformément au paradoxe de d'Alembert. Je ne sais si l'expérience a fourni des données sur ce point.
>
> Veuillez donc je vous prie, n'attacher aucune importance à ma lettre précédente et croire à mes sentiments très amicaux
>
> J. Hadamard

In 1907 Levi-Civita published another fundamental work about wake and resistance laws, which solved many problems in plane hydraulics and gave a great impulse to analytical studies of hydrodynamics [Levi-Civita, 1907a]. The method used by Levi-Civita is based on complex analysis and was already contained *in nuce* in some works by Hermann von Helmholtz (1821–1894) and Gustav Kirchhoff (1824–1887). Levi-Civita's method allows one to deduce the general integral of the rectilinear and uniform motion with wake of a given body with any shape (see [Supino, 1975]). In technical terms, to any analytic function, supposed to be regular in the inside of a circle and real on each of its diameters, there corresponds an irrotational motion with wake.

About his method, which overcame many difficulties contained in the works of Helmholtz and Kirchhoff, Levi-Civita was proud to write to Villat on June 19, 1911[5]:

> Je me réjouis infiniment que le perfectionnement de la méthode de Helmholtz–Kirchhoff signalé par moi, ait donné l'essor à bien des recherches savantes.

As Umberto Cisotti (1882–1946)—one of Levi-Civita's students—pointed out [Cisotti, 1912, 493]:

> But the method so extraordinarily developed [by Levi-Civita] allowed other authors to solve many other hydrodynamic problems, by always reaching concrete results with precision, evidence and elegance.

Marcel Brillouin (1854–1948) and particularly Villat in his *Thèse* [1911] improved Levi-Civita's method and made it more easily applicable to concrete cases.

In 1907 Levi-Civita published another influential paper on hydrodynamics, this one concerning progressive permanent waves in a canal with horizontal bottom [Levi-Civita, 1907b]. Here "progressive" means that the motion appears stationary to an observer moving together with the apparent translation of the fluid. Levi-Civita considered motions in the vertical plane, so that gravity intervenes as a fundamental force. By means of results and theorems belonging to the theory of functions of a complex variable, the original problem is reduced to the solution of a particular differential equation related to a unique holomorphic function.

In a subsequent paper, Levi-Civita went on to study permanent waves in a canal with horizontal bottom under some particular physical conditions [Levi-Civita, 1912]. In particular, he proved the so-

[5] Levi-Civita's letters to Villat are contained in *Dossier Villat*, "Archives de l'Académie des Sciences," in Paris.

called generalized Stokes–Rayleigh theorem (the transport of fluid mass increases without limit with time) and deduced a new formula for the kinetic energy of waves.

In some lectures given at the University of Barcelona, Levi-Civita [1922a] gave a systematic treatment of the theory of canal waves, making use of the physical concept of a *wave-motion phenomenon*. The theory of canal waves had been the subject of investigation in some classic works of George Biddell Airy (1801–1892), George Gabriel Stokes (1819–1903), and John William Strutt, Lord Rayleigh (1842–1919). But a fundamental problem remained unsolved—the determination of periodic and irrotational permanent waves. In 1802 F. von Gerstner (1756–1823) gave the solution for periodic and permanent rotational waves moving in canals of infinite depth. Airy, Stokes, and Rayleigh had approached the case of irrotational waves by applying Gerstner's results and limiting the analysis to first approximations. As Marie-Louise Dubreil-Jacotin (1905–1972)[6] remarked [1934, 217–218], "l'existence de ces ondes [irrotationnelles] n'était pas démontrée et Lord Rayleigh en douta même un moment. C'est en 1925 que M. Levi-Civita dans un Mémoire fondamental [Levi-Civita, 1925b] établit l'existence de l'onde irrotationnelle dans le cas de la profondeur infinie." In fact, as Levi-Civita [1925b, 264] pointed out in his own work: "J'y développe tous les détails de concept et de calcul qui permettent de caractériser nettement les ondes périodiques irrotationnelles permanentes, c'est-à-dire pouvant se propager sans altération de forme à la surface d'un liquide de profondeur infinie." In his classical treatise on hydrodynamics, Horace Lamb (1849–1934) [1932, 420] observed that Levi-Civita's proof of the existence of irrotational waves "puts an end to an historical controversy."

Levi-Civita's work influenced many authors, who extended his procedure to more general cases. Lamb quoted the extension of Levi-Civita's method to the case of canals of finite depth by Dirk Jan Struik (1894–2000) [Struik, 1925], one of Levi-Civita's Rockefeller students.[7] N.E. Kotchine (1901–1944) also employed Levi-Civita's procedure in studying irrotational waves on the separating surface between two fluids with different densities.

Dubreil-Jacotin, who was Villat's student but also Levi-Civita's student in Rome, proved [1934] the existence of infinite rotational waves, which included the irrotational wave—whose existence had already been shown by Levi-Civita [1925a, 1925b]—and Gerstner's wave as particular cases. In some notes [Dubreil-Jacotin, 1932, 1935] communicated by Levi-Civita to the Academy of Lincei, she also studied permanent waves in heterogeneous fluids. As Yih [1967, 539] observed in the introduction to a paper written in 1967, "the equations governing steady two-dimensional flows were given by Madame Dubreil-Jacotin for an incompressible fluid of variable density [1935, p. 345, equation (B)] and for an ideal gas of variable entropy [1935, p. 346, equation (b)]. These equations were later rediscovered by Prof. Long [1953a, 1953b], and have been effectively and fruitfully utilized by him in his excellent studies of atmospheric waves."

If water is poured into the beginning of a canal, then a wave will propagate—it is called a *solitary wave*. Many students of Levi-Civita studied this wave phenomenon, among them, Cisotti, Luigi Sante Da Rios (1881–1965) (see [Ricca, 1991, 1996]) and some other students, such as Struik, Dubreil-Jacotin, and Alexander Weinstein (1897–1979).[8]

[6] Paul Dubreil (1904–1994), who was in Rome as a Rockefeller student, gives a good description of his Roman period, including his wife's contributions to hydrodynamics, in [Dubreil, 1983].

[7] Levi-Civita's testimonial on Struik's research is reported in Appendix A.

[8] About Weinstein and his research activity under Levi-Civita's supervision in Rome see Appendix B.

The asymptotic form of the Newtonian potential for slender tubes is a problem connected to hydrodynamics, which was studied by Levi-Civita and by some of his students during the first decade of the 20th century. In the twenties and thirties Levi-Civita analyzed Saturnian rings and vortices as well as vortex filament dynamics, using some results due to Da Rios. As Ricca points out [Ricca, 1996, 266]:

> The work of Da Rios and Levi-Civita on vortex filament motion and asymptotic potential theory spanned a period of almost 30 years, from 1906 to 1933, and represents one of the major contributions to three-dimensional vortex filament dynamics. [...]
>
> In the light of modern developments of non-linear dynamics and vorticity, their work strikes for modernity and depth of results.

1.3. Tensor calculus

Elie Cartan (1869–1951) wrote in his obituary of Levi-Civita communicated to the Academy of Sciences of Paris on September 14, 1942 [Cartan, 1942, 234]:

> Mais ce qui a permis à la renommée de notre Confrère de déborder le cercle des spécialistes c'est le rôle qu'il a joué dans l'histoire du calcul différentiel absolu, ce sont les nombreuses applications qu'il en a données.

Ricci-Curbastro elaborated tensor calculus in the years 1885 to 1895. The principal sources were the theory of algebraic forms and the theory of invariants on Riemannian manifolds as developed by Bernhard Riemann (1826–1866), Rudolph Lipschitz (1832–1903), and Elwin Bruno Christoffel (1829–1900). Although these authors obtained "relevant" results, "their methods do not always appear evident"—as Ricci pointed out [Ricci-Curbastro, 1884, 140]. His "absolute differential calculus" was intended to simplify their methods (see [Dell'Aglio, 1996] and [Reich, 1994]).

In 1899 Felix Klein (1849–1925) met Levi-Civita in Padua and asked him to publish in his journal, *Mathematische Annalen*, an organic and systematic account of tensor calculus.[9] Levi-Civita responded to this request by writing—together with Ricci-Curbastro—the well-known paper "Méthodes de calcul différentiel absolu et leurs applications" [Levi-Civita and Ricci-Curbastro, 1900]. Levi-Civita's contribution was connected to the applications of tensor calculus to mathematical physics, in particular to analytical mechanics, binary potentials, and Lie groups of motions.

Levi-Civita had already used tensor calculus in his dissertation, which was written under the direction of Ricci-Curbastro and published with some minor changes in [Levi-Civita, 1893–1894]. By putting together Ricci-Curbastro's algorithm with some results from Lie's theory of transformation groups, Levi-Civita extended the theory of absolute invariants to more general cases than those considered by Ricci-Curbastro. Some years later, starting from some work of Lipschitz, Paul Appell (1855–1930), and Painlevé, Levi-Civita [1896] studied the transformation laws between two systems of dynamic equations. His aim was to derive a change of coordinates that transformed an Euler–Lagrange system to another one that was simpler to integrate. In the case where no external force acted on the system, he solved the general problem by reducing it to a geodetic representation on a Riemannian manifold.

This contribution is mentioned in the "Rapport sur le travaux scientifiques" of Levi-Civita, dated May 29, 1911, written by H. Léauté in order to appoint Levi-Civita "membre correspondant de l'Académie des Sciences de Paris"[10]:

[9] The event is told in a letter (March 30, 1899) by Levi-Civita to Arnold Sommerfeld, which is kept in the "Archives of the Deutsches Museum." We thank Dr. Wilhelm Füssi, who sent us a copy of the letter.

[10] In *Dossier Levi-Civita*, "Archives de l'Académie des Sciences" of Paris.

> M. Levi-Civita qui, dans la première partie de sa carrière, avait montré, aussi bien en Analyse qu'en Géométrie, ses qualités brillantes de Mathématicien, fut amené ensuite à aborder certaines questions importantes de Mécanique rationnelle. Il publia tout d'abord toute une série de Mémoires se rattachant à l'étude des équations de la Dynamique et dont le point de départ était un problème, posé par nos Confrères MM. Appell et Painlevé, qui en avaient montré tout l'intérêt [Levi-Civita, 1896]. Ce problème peut s'énoncer comme il suit :
>
> Deux systèmes matériels ayant même degré de liberté mais des structures différentes, c'est-à-dire des expressions différentes pour la force vive, ne suivent pas, en général, les mêmes lois de mouvement. Sous quelles conditions y aura-t-il coïncidence dans la succession des positions prises par le mobile indépendamment du temps ? Cette question de l'identité des trajectoires est résolue dans un cas particulier et l'Auteur forme tous les types de force vive qui y correspondent.

Léauté remarked that "M. Levi-Civita obtient une grande simplification par l'emploi des méthodes de calcul indiquées par M. Ricci."

Such methods were also used by Levi-Civita in his paper on binary potentials [Levi-Civita, 1899], whose starting point was Riemann's *Commentatio Mathematica* [Riemann, 1861]. Here Levi-Civita applied tools belonging to many areas—theory of infinitesimal transformation groups, potential theory, differential geometry, and tensor calculus—in order to classify all the potentials that depend on two variables only.

Tensor calculus is very useful in applications in analysis, geometry, and especially mathematical physics. In fact, tensorial equations are invariants; that is to say, they are not affected by changes of coordinates. That is why tensors are particularly useful for expressing geometrical, analytical, and physical relations that are independent of the particular coordinate system that is applied. In justifying their method, Ricci-Curbastro and Levi-Civita wrote [Levi-Civita and Ricci-Curbastro, 1900, 481]:

> M. Poincaré a écrit que dans les Sciences mathématiques *une bonne notation a la même importance philosophique qu'une bonne classification dans les Sciences naturelles*. Evidemment, et même avec plus de raison, on peut en dire autant des méthodes, car c'est bien de leur choix que dépend la possibilité de *forcer* [...] *une multitude de faits sans aucun lien apparent à se grouper suivant leurs affinités naturelles.*

In 1917 Levi-Civita [1917a] introduced the new concept of "parallel translation," which made tensor calculus more intuitive and easier to apply. In his paper Levi-Civita defined the concept of *parallelism* on a Riemannian manifold using a new formulation of the inertial law. As in the Euclidean plane, he considered a geodetic line and a vector moving in space, such that its point of application always belongs to the geodetic. Therefore, the vector moves as a parallel if it always makes the same angle with the tangent to the geodetic. In particular, a tangent to a geodetic will be parallel to itself if it moves along the geodetic; from this definition it is possible to deduce the analytic expression of a geodetic line on a Riemannian manifold, which physically represents an inertial motion on the manifold.

Finally, thanks to his concept of parallel translation, Levi-Civita was able to give a geometrical interpretation of the Riemannian tensor of curvature by introducing an appropriate parallelogram on the manifold. In his *Lectures* [1925c] on the absolute differential calculus he returned to this subject and more generally to the geometrical consequences of parallel translation. Before his treatise, other books on certain mathematical aspects of tensor calculus had appeared.[11] One of the most interesting was published by J. Edmund Wright (1879–1910) in 1908; it contains a treatment of dynamical equations and their transformation laws based on tensorial methods [Wright, 1908]. It was much appreciated by Levi-

[11] See [Juvet, 1922; Galbrun, 1923; Marais, 1923; Schouten, 1924].

Civita himself, as one can infer by reading the following letter from Wright to Levi-Civita (September 11, 1908):

> Dear Sir,
>
> Thank you very much for your kindness in sending me your papers, and also for your most encouraging letter about my small book [Wright, 1908].
>
> I am hoping to write some day a larger book on the same subject, and should therefore be most grateful if you should at any time care to make any comments or criticisms. I have just a small paper on "Corresponding Dynamical Systems." I take the liberty of sending it to you in the hope that you may think it worthy of being published in the Annali di Math.
>
> I hope you will forgive me for this troubling you.
>
> With many thanks
>
> Yours sincerely

Many books on tensor calculus were published after the formulation of general relativity; but their aim was to help physicists understand the formalism used by Einstein for deducing his gravitational field equations. In contrast, the *Lectures* of Levi-Civita were addressed to mathematicians and showed the tensor calculus from a different point of view, where geometry played the most important role. Levi-Civita's approach could also be appreciated by geometers working in the 19th-century tradition, since it allowed rewriting of many classical concepts of differential geometry in a tensorial form. For example, Luigi Bianchi (1856–1928) wrote a paper on the parallel translation of Levi-Civita [Bianchi, 1922] and published his ideas in *Appendix* to the second volume of his celebrated *Lectures on Differential Geometry* [Bianchi, 1923].

Levi-Civita's work on parallel translation and its *geometric* applications to analysis and mathematical physics inspired Cartan's works on Riemannian manifolds. As Elie Cartan (1869–1951) himself wrote in his commemoration of Levi-Civita [Cartan, 1942, 234]:

> Il était réservé à Levi-Civita de lui apporter un dernier perfectionnement [du calcul différentiel absolu] par la découverte en 1917 de la notion du transport parallèle. En rendant plus intuitives les notions fondamentales du calcul différentiel absolu, elle faisait entrer une théorie, jusqu'alors purement analytique, dans le domaine de la Géométrie. Il en résulta des répercussions profondes sur le développement de la Géométrie elle-même, à laquelle la découverte de Levi-Civita donna un nouvel essor, comparable par son ampleur à celui que lui avait imprimé près d'un demi-siècle auparavant le célèbre Programme d'Erlangen de Felix Klein. Une théorie générale nouvelle des transports parallèles et des connexions, susceptible de fournir de nouveaux schémas géométriques aux physiciens, allait s'édifier par les travaux de H. Weyl, J.A. Schouten, O. Veblen, L. Eisenhart, E. Cartan. Par son rôle de précurseur dans cette ample floraison de la Géométrie différentielle, le nom de notre confrère Levi-Civita est assuré d'une place non moins éminente dans l'histoire de la Géométrie que dans l'histoire de l'Analyse, de la Mécanique et de la Physique mathématique.

The following letter of Emile Borel (1871–1956) (February 17, 1923) shows that Levi-Civita's *Lectures* on tensor calculus were considered a necessary instrument for mathematicians as well as a classical geometric work:

> Mon cher Collègue et ami,
>
> La maison Gauthier Villars a l'intention de publier une collection de Physique mathématique et a demandé à M. Brillouin et à moi-même de se charger de la diriger. Nous serions très désireux de pouvoir publier le plus tôt possible un ouvrage de vous. Comme nous savons que vous êtes fort occupé par des travaux originaux importants, nous permettons de vous suggérer qu'il serait fort intéressant et utile de réimprimer votre mémoire fondamental des Mathematische Annalen (en collaboration avec M. Ricci) [Levi-Civita and Ricci-Curbastro, 1900], qui est difficile à trouver et auquel on doit se référer constamment. Si l'on joignait la traduction de vos notes des Lincei sur le déplacement parallèle [Levi-Civita, 1917a] et sur la statique einsteinienne [Levi-Civita, 1917b], on aurait déjà, sous un faible volume, un ouvrage extrêmement précieux. Bien entendu, si vous entrevoyez la possibilité de rédiger un ouvrage plus étendu, nous en serions très heureux ; mais, si vous n'envisagez pas une telle rédaction comme prochaine, il nous semble que le programme ci-dessus, immédiatement réalisable, donnerait dès à présent des résultats extrêmement utiles au progrès de la science.

The answer is contained in the following (undated) rough copy written by Levi-Civita:

Très honoré Coll. et cher Ami,

Je vous remercie infiniment pour votre proposition si aimable et si flatteuse que vous me faites dans votre lettre du 17 de ce mois. Il me serait sans doute fort agréable de collaborer dans une forme fraîche et inédite à la collection que vous allez diriger avec M. Brillouin et qui aura évidemment le plus substantiel et le plus brillant des succès. Malheureusement ma capacité d'auteur est très faible et je suis maintenant absorbé (non, comme vous l'admettez avec grande bienveillance, par des recherches originales), mais par la préparation (avec M. Amaldi) d'un cours de mécanique rat., dont le premier volume va paraître dans quelques jours [Amaldi and Levi-Civita, 1923]. Je dois partant me contenter d'accepter avec reconnaissance votre offre de reproduire quelques travaux antérieurs.

A ce propos toutefois, je crois avant tout nécessaire de vous prévenir que j'ai tenu il y a deux ans ici à Rome (et je répète sans grandes modifications cette année) un cours sur le Calc. diff. absolu, en simplifiant pas mal l'apparat formel, [avec] l'usage syst. de la notion de parallélisme. Ce cours a été recueilli par un de mes auditeurs (M. Persico) et sa rédaction (que j'ai revue) va être imprimée. [Crossed out: L'éditeur, qui est un débutant, a été lui aussi un des auditeurs.] On compte de commencer bientôt et de faire paraître le petit volume (quelques 200 ou 250 pages) avant la fin de l'année [Levi-Civita, 1925c].

M. Juvet (de la part de l'éditeur Blanchard) m'a demandé l'autorisation de traduire ce volume en français. Pour ma part j'avais immédiatement consenti avec plaisir, mais les éditeurs ne se sont pas réussi à s'accorder (du moins jusqu'à présent). M. Juvet pense que cela dépend de mon éditeur, M. Stock. Je n'en puis rien dire, tout en ayant l'impression que M. Stock, qui est un débutant, soit plein d'initiative et d'enthousiasme.

Si vous croyez qu'il puisse vous convenir de publier la traduction du cours susdit, je me tiens entièrement à votre disposition pour tous les renseignements et pour toutes les demandes que vous désirez. Si d'autre parte vous croyez préférable de maintenir le plan de votre lettre, je me chargerais très volontiers d'obtenir l'autorisation de M. Ricci. Quant à la traduction il est évidemment préférable qu'elle soit confiée à un français. Peut-être, on pourra penser à M. Barbier Professeur au Lycée Chateaubriand.

"Le petit volume" of his lectures was printed two years later [Levi-Civita, 1925c] by Stock and was never translated into French. Borel seemed very interested in the possible French translation of Levi-Civita's lectures on tensor calculus and wrote to Levi-Civita on February 26:

Mon cher ami,

Je vous remercie de votre lettre ; votre suggestion concernant la possibilité de publier la traduction de votre cours me parait très intéressante et j'ai demandé à M. [Traupecher] Directeur de la maison Gauthier Villars, d'écrire à ce sujet à votre éditeur ; j'espère qu'une entente sera possible entre eux ; si elle ne l'était pas nous reviendrions à mon premier projet. (. . .)

Unfortunately the two publishers did not come to an agreement, as Borel wrote to Levi-Civita on March, 8:

Mon cher ami

M. Traupecher, directeur de la Maison Gauthier Villars a reçu la réponse de M. Stock à la lettre qu'il lui avait écrite sur ma demande. Malheureusement, il considère que les conditions posées par M. Stock sont trop dures ; elles dépassent ce qu'il donnerait à l'auteur d'un livre original en français. Dans ces conditions, il me semble que le mieux, si cela vous convient, serait de revenir à ma première proposition. Pouvez vous, comme vous me l'aviez aimablement proposé, vous assurer de l'agrément de M. Ricci? (. . .)

Borel came back to the French translation of Levi-Civita's memoirs again in the following letter (March, 28):

Mon cher ami

Ma réponse à votre lettre du 16 Mars a été retardée parce que j'ai été atteint de grippe et obligé de garder le lit plusieurs jours. Je vais maintenant aller prendre un peu de repos dans le sud-ouest de la France.

Mais je voudrais vous remercier de la part aussi de M. Brillouin, pour votre aimable acceptation de principe, de nous donner un petit livre pour notre Collection. Ce livre comprendrait tout d'abord votre Mémoire des Mathematische Annalen ; il comprendrait aussi les traductions de vos Notes sur le parallélisme et sur la Statique einsteinienne.

Au sujet de ces Notes, veuillez me dire s'il vous est plus commode de nous envoyer la traduction française ou le texte italien que nous ferions traduire ici. D'autre part, possédez vous encore un tirage à part des Math. Annalen ou serons nous obligés de faire

> copier le Mémoire sur l'exemplaire d'une bibliothèque. Dès que vous m'aurez répondu sur ces questions matérielles, j'en ferai part à Gauthier Villars et lui dirai de vous envoyer un projet de traiter que vous examinerez et que vous accepterez si vous le jugez convenable. (...)

Borel's attempt at publishing Levi-Civita's works failed. In 1923 the French translation of the 1900 memoir of Ricci-Curbastro and Levi-Civita on tensor calculus was indeed published by Blanchard in the *Collection de monographies scientifiques étrangères publiées sous la direction de M.G. Juvet.*

In 1927 an English edition of Levi-Civita's lectures on tensor calculus was printed—several English editions appeared from 1929 onward—while the German edition was published the next year (1928). We notice that the English book is not a simple translation of the Italian text. In fact, it contains a Part III: *Physical Applications*, which is not in the Italian treatise. Part III deals with mechanics and geometrical optics in relation to a four-dimensional world—according to Einstein's Special Theory of Relativity—and the gravitational equations of General Relativity. As Levi-Civita pointed out in the preface of the book:

> The present volume contains a complete translation, made in consequence of a suggestion by my eminent friend, Professor E.T. Whittaker, F.R.S., of the Italian text of my *Lezioni di calcolo differenziale assoluto*. Two new chapters have been added, which are intended to exhibit the fundamental principles of Einstein's General Theory of Relativity (including, of course, as a limiting case, the so-called Special or Restricted Theory) as an application of the Absolute Calculus.

The *new* "application of the Absolute Calculus" was very important and was greatly appreciated; in this connection, we report what Cisotti wrote in a letter to Levi-Civita dated January 11, 1927:

> Very Dear Friend,
>
> I received, by your English Publisher, the splendid volume of your "The absolute differential calculus." I really admire its exterior aspect but mainly its content; especially, the Part III—about applications—seems to me to have the most perfect form. (...)[12]

1.4. General relativity

The concept of a parallel translation is fundamental for developing and formulating certain ideas about relativity. Indeed Levi-Civita dedicated himself to general relativity for many years and published about 40 notes on the subject, which contain substantial results. An essential contribution to general relativity can also be found in the correspondence between Albert Einstein (1879–1955) and Levi-Civita from March to May 1915 (see [Cattani and De Maria, 1989]).

Let us establish some basic background. In 1912, thanks to the advice of his friend Marcel Grossmann (1878–1936)—professor of mathematics at the Polytechnic of Zurich—Einstein read the works of Gauss and Riemann in differential geometry and began studying the tensor calculus of Ricci-Curbastro and Levi-Civita. In 1913 the first result of his collaboration with Grossmann appeared—the paper *Entwurf* [Einstein and Grossmann, 1913]. Nevertheless the crucial problem of the gravitational field equations was unsolved—in this paper the field equations are not considered generally covariant, but their invariant group is restricted to linear transformations only. This point of view led Einstein and Grossmann to make

[12] Carissimo Amico,

Ho ricevuto dal tuo Editore inglese, il magnifico volume del tuo "The absolute differential calculus" e sono ammirato veramente dell'esteriore e soprattutto dell'interiore, specialmente la Parte III, quella delle applicazioni, che mi pare abbia raggiunto la forma più desiderabile.

improbable physical assumptions. In some subsequent papers, Einstein and Grossmann attempted to justify the gravitational field equations derived in the *Entwurf* paper by means of variational principles.

The difficulties connected with the *right* expression of the gravitational equations are the main subject of the correspondence between Einstein and Levi-Civita. Einstein was always grateful to Levi-Civita for his interest in general relativity; on April 2, 1915 Einstein wrote to Levi-Civita:

> So interesting a correspondence I have never experienced. You should see how I am happy when I receive your letters.[13]

In his letters, Levi-Civita noticed an error in the proof of the *Entwurf* paper, whose consequences involve the covariant properties of the gravitation tensor. Einstein tried several times to refute Levi-Civita's objections. In a letter dated March 5, 1915, he wrote:

> Dear Colleague,
>
> I am very happy that you are so interested in my work. You can image how uncommon it is that someone is deeply interested in this subject with an independent and critical mind. [. . .]
>
> As I noticed that you criticized the most important proof in my theory, obtained from streams of sweat, I was not a little frightened since I know that you handle such mathematical questions much better than I. However, after careful deliberation I think that my proof can be upheld.[14]

And on April 21, 1915 Einstein wrote to Levi-Civita that he hoped to persuade him of the validity of "his Theorem," since—according to him—Levi-Civita's objection could be overcome.

The epistolary controversy between Einstein and Levi-Civita went on until early May. On May 5, 1915, Einstein had to admit that his proof was "defective." Einstein started a hard but fruitful period of his life. Finally, in a note communicated to the *Preussische Akademie der Wissenschaften* on November 25, 1915, Einstein obtained the correct expression for the gravitational field equations. Just after the publication of the right gravitational equations, Levi-Civita published many papers on General Relativity—on Einstein's gravitational tensor [1917c] and on Einsteinian statics [1917b, 1917–1919].

"The gravitational equations represent a real triumph of Ricci-Curbastro's mathematical methods," as Einstein himself observed. Tensor calculus was then indispensable in the theory of relativity. By contrast, Ricci-Curbastro's contemporaries did not find that the difficulties of his method could be justified by results, which could also be deduced by classical and simpler procedures. In Italy tensor calculus only became established among mathematicians after the Second World War, when tensors were used in elasticity, magnetism, theory of heat, electrodynamics, and general relativity.

Thus relativistic theories did not spread in Italy as rapidly as one might expect today, even though Levi-Civita and some of his students were interested in questions related to general relativity. It was only in 1921, when Einstein gave some lectures in Italy, that his theory became popular and, still, more popular among mathematicians than among physicists. Italian physicists were indeed mainly interested

---

13 "Eine so interessante Korrespondenz habe ich noch nicht erlebt. Sie sollten sehen, wie ich mich immer auf Ihre Briefe freue."

14 Hoch geehrter Herr Kollege!

Sie erweisen mir damit, dass Sie sich so genau mit meiner Arbeit befassen, eine grosse Freude. Sie können sich denken, wie selten sich jemand eingehend mit dieser Sache beschäftigt, der ihr selbständig und kritisch gegenübersteht. [. . .]

Als ich sah, dass Sie Ihren Angriff gegen den wichtigsten, mit Strömen von Schweiss erkauften Beweis der Theorie richten, erschrak ich nicht wenig, zumal ich weiss, dass Sie diese mathematischen Dinge weit besser beherrschen als ich. Noch eingehender Überlegung glaube ich aber doch, meinen Beweis aufrecht erhalten zu können.

in experimental physics, while a few mathematicians, in primis Levi-Civita, studied theoretical physics. Moreover, in the Italian universities vector calculus was much more appreciated and studied than tensor calculus. Since general relativity was formalized by means of tensors and relations between tensors, the preeminence of *vectorialists* over *tensorialists* was another reason that general relativity did not spread in Italy immediately.

Levi-Civita engaged in popularizing the theory of relativity in Italy—and not just in Italy.[15] In 1921, in Barcelona, he delivered popular lectures on classical and relativistic mechanics, whose Italian and German translations were published in 1924 [Levi-Civita, 1922a]. To appreciated his role as an expert on the theory of relativity in Italy, consider the following letter from Augusto Righi (1850–1920) to Levi-Civita—undated but probably written between the end of 1919 and the beginning of 1920:

> Dear Professor,
>
> The International Committee of Physics founded by Solvay will meet in Bruxelles soon.[16] By a lucky chance, some months ago I was asked to enter the Committee. We shall have to organize the summer meeting, to decide the subjects to deal and to choose the physicists to be invited.
>
> It is probable that Relativity will be one of the subjects. In such a case, I thought that you must be the first to be invited. Even if I am the latest arrival in the Committee, I intend to propose you. Do you authorize me to do so?
>
> When you reply, I would prefer that you tell me, or that you repeat, the biographical information about Einstein that you have already mentioned—whether he was born or lived in Italy, and how and where he was, and where he is now. A journalist friend asked me for such information and I would like to give it to him.
>
> I hope that I can soon speak with you in Rome about a little nuisance due to the memoir that you read a year ago, and first of all I would like to speak with you about Einstein's theory, of which I am becoming enthusiastic. I thought a lot about what you had told me that day in December at Lincei. The good seed is bearing fruit!
>
> I am looking forward to your answer. Kind regards. I would be in Rome now, if I were not under an avalanche of students who must be set on the right road!
>
> A. Righi[17]

The answer of Levi-Civita was immediate. In a letter dated February 11, 1920, he declined the invitation to participate in the Solvay Congress, since he feared to give a lecture not interesting enough for

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[15] See [Caparrini, 1998; De Maria, 1987].

[16] Righi refers to the third "Conseil de Physique Solvay" (Brussels, April 1–6, 1921) on the subject: "Atoms and electrons" (see [Marage and Wallenborn, 1995]).

[17] Caro professore

Presto dovrebbe riunirsi a Bruxelles il Comitato Internazionale di Fisica creato dal Sig. Solvay, di cui, non so per quale caso fortunato, sono stato chiamato a fare parte qualche mese fa. Si dovrà preparare la riunione estiva, fissare gli argomenti da trattare e scegliere i fisici da invitare.

E' probabile che fra gli argomenti vi sia la Relatività. Quindi senz'altro ho pensato che il primo da invitare sarebbe lei. Per quanto io sia l'ultimo arrivato in quel Comitato, io intendo proporre d'invitare lei. Mi ci autorizza?

Rispondendomi vorrei mi dicesse, o meglio mi ripetesse, le informazioni biografiche che mi accennò relative a Einstein. Cioè se è nato o vissuto molto in Italia, e come, dove fu, e dove è adesso. Un amico giornalista mi ha chiesto queste notizie e desidererei potergliele dare.

Spero di poter parlarle presto a Roma, di una piccola seccatura procuratami dalla Memoria che le feci leggere un anno fa, e sopra tutto vorrei conversare con lei sulla teoria d'Einstein, della quale sto diventando entusiasta. Ho ponderato incessantemente su quanto Ella ebbe a dirmi quel giorno di dicembre ai Lincei. Il buon seme sta fruttificando!

In attesa di riscontro la saluto cordialmente. Sarei già a Roma se non fossi soffocato sotto una valanga di studenti e di laureandi, che bisogna instradare!

A. Righi

an audience of physicists. Moreover, he referred to the biographical notices about Einstein and proudly added:

> The new relativity was built between 1913 and 1915 a little gropingly. As you well know, the essential analytical instrument of this theory is the absolute differential calculus of Ricci. Einstein openly recognizes this fact in the introduction to the two notes appeared in Sitzungsber. of Berlin on November 4th and 11th, 1915 [Einstein, 1915a, 1915b], where the differential equations of the theory are given in a definitive way.[18]

Abroad Levi-Civita's popular and scientific works on relativity were widely known and appreciated. This is demonstrated by Mittag-Leffler's invitation to Levi-Civita to write a paper on the theory of relativity for his journal *Acta Mathematica*, where the mathematical point of view was to be accented (January 4, 1922):

> Mon cher Ami,
>
> J'ai une proposition à vous faire. Voudriez-vous m'écrire une appréciation ou un compte rendu détaillé sur Einstein au point de vue mathématique pour être publié dans les Acta Mathematica ? Vous êtes, je crois, le premier qui a ouvert la discussion sur la théorie de la relativité au point de vue mathématique et je ne connais personne plus compétent que vous d'écrire un tel mémoire que je me permets de vous demander.
>
> Avec mes meilleurs voeux pour le nouvel an
>
> Votre ami dévoué
>
> Mittag-Leffler

Levi-Civita accepted Mittag-Leffler's request enthusiastically; however, he was engaged—together with Ugo Amaldi (1875–1957)—in writing a treatise on rational mechanics [Amaldi and Levi-Civita, 1923] and asked for more time (January 9, 1922):

> Monsieur et cher Maître,
>
> Je suis charmé de votre idée ainsi que de votre aimable invitation si flatteuse pour moi. Je l'accepte, avec autant de plaisir que de reconnaissance pour la considération bienveillante dont vous m'honorez, pourvu toutefois qu'il ne vous gêne pas d'attendre quelques mois.
>
> Pendant cette année solaire je me trouve presque complètement absorbé par les cours (j'en ai deux) et par la préparation d'un traité de mécanique rationnelle (en collaboration avec M. Amaldi) [Amaldi and Levi-Civita, 1923], de façon qu'il ne me serait possible de soigner l'article en question avec la fraîcheur d'esprit et la pondération qu'il exige de ma part pour avoir quelque chance de réussir selon votre intention et votre initiative si engageante. Il faudrait donc que j'y songeais pendant les vacances d'été.
>
> Il va d'ailleurs sans dire que je vous resterai également très obligé d'avoir pensé à moi, si, à cause de ma réponse dilatoire, vous adresserez à quelque collègue ayant une vitesse de travail supérieure à la mienne, qui est malheureusement très faible.
>
> Agréez, illustre et cher maître, mes meilleurs voeux pour votre prospérité et les sentiments de mon admiration dévouée.
>
> T. Levi-Civita

Mittag-Leffler answered on January 16, 1922:

> Mon cher Ami,
>
> Je vous remercie cordialement de votre lettre ainsi que de votre promesse aimable de m'écrire sur la question de la relativité. Ce qui manque dans toutes les publications qu'on publie sans cesse, c'est toujours un fondement solide de mathématiques. Sans un tel fondement on ne viendra jamais à un résultat satisfaisant. J'espère de recevoir de vous un tel fondement. Weierstrass et Riemann le

[18] La nuova relatività fu costruita un po' a tentoni fra il 1913 e il 1915. Come Ella ben sa, lo strumento analitico essenziale di questa teoria è il calcolo differenziale assoluto del Ricci. Einstein lo riconosce nel modo più esplicito nell'introduzione alle due note del 4 e 11 Novembre 1915 dei Sitzungsber. di Berlino in cui sono date in forma definitiva le equazioni differenziali che traducono la teoria.

cherchaient tous les deux mais sans aboutir à un résultat final satisfaisant. Poincaré de même. Il l'aurait trouvé si sa mort n'était pas venue d'une manière si inattendue.

J'espère et je crois que vous êtes l'homme de résoudre le problème et je me ferais un honneur de donner une telle publicité à votre travail qu'il méritera sans doute.

En attendant, veuillez agréer l'expression de ma haute considération et de mon dévouement sincère.

G. Mittag-Leffler

In a letter to Mittag-Leffler (April 8, 1922), Levi-Civita communicated the title of his forthcoming paper in *Acta Mathematica*: "Sur la théorie des perturbations en relativité générale." On October 18, 1923, Mittag-Leffler wrote to Levi-Civita:

J'attends encore avec beaucoup d'intérêt l'article concernant l'idée de relativité que vous avez eu l'amabilité de me promettre pour mes Acta Mathematica, mais j'ai aujourd'hui encore une proposition à vous faire. Dans chaque tome des Acta qui paraîtra dorénavant je voudrais, si possible, publier des rapports bibliographiques dans le genre des notices de travaux qu'on fait en France pour les concours de l'Institut ou dans celui par H. Poincaré « Analyse de ses travaux scientifiques », publié dans le tome 38 des Acta, dont je vous envoie un tirage à part sous pli séparé. Je m'adresserai avec ma demande à des géomètres dans les pays différents que je considère comme les tous premiers à notre époque. En effet, je vous serais fort reconnaissant, si je pouvais attendre une telle notice par vous, et je suis tout à fait convaincu que vous rendriez par là un grand service aux sciences mathématiques.

In his immediate answer (October 22, 1923) Levi-Civita wrote:

J'avoue tout d'abord mon tort et mon vif regret de n'avoir pas encore tenu mon engagement de vous fournir un article pour les Acta sur les équations de la mécanique céleste au point de vue de la relativité.

La raison en est que j'ai été (et je le serai pour quelque temps encore) absorbé par la préparation (en collaboration avec M. Amaldi) de notre traité de mécanique rationnelle. [. . .]

J'accepte, cela va sans dire, avec grand plaisir et sincère reconnaissance, l'offerte d'écrire un rapport sur mes propres travaux ; mais je considère mon strict devoir de vous remettre auparavant l'écrit incriminé.

However, after the publication of the book on rational mechanics [Amaldi and Levi-Civita, 1923], Levi-Civita was engaged in writing his *Lectures* on absolute differential calculus published in 1925 [Levi-Civita, 1925c]. In 1927 Mittag-Leffler's death put an end to all his former requests and Levi-Civita never published his paper on the theory of perturbations in general relativity or his "Analyse de ses travaux scientifiques."

2. Levi-Civita at the University of Rome

2.1. Rome: the chief town of Italian mathematics

Levi-Civita moved to the University of Rome in 1918. In 1909, just after the death of Valentino Cerruti (1850–1909), Guido Castelnuovo (1865–1952) had tried unsuccessfully to persuade Levi-Civita to come to Rome. In the following years and after the First World War other attempts were probably made, in spite of Levi-Civita's pacifist and socialist ideas that were so much in contrast with the interventionist convictions of Vito Volterra (1860–1940). When the war was over, Roman mathematicians and first of all Volterra—at that time, the dean of the Faculty of Science—wanted to strengthen teaching and research in mathematics. With this aim, Levi-Civita was nominated Professor of Higher Analysis at the University of Rome. Two years later he became Professor of Rational Mechanics.

Many other new professors came to the University of Rome in the interwar period. To the "Roman group" the following mathematicians belonged (the years of their arrival in Rome appear in parentheses): Guido Castelnuovo (1892), Vito Volterra (1901), Tullio Levi-Civita (1918), Federigo Enriques (1921), Giuseppe Bagnera (1922), Francesco Severi (1922), Ugo Amaldi (1924), Enrico Bompiani (1927), Mauro Picone (1932), Gaetano Scorza (1935), Antonio Signorini, and Leonida Tonelli (1939). Notice that the most important professors of algebraic geometry in Italy—Castelnuovo, Enriques, and Severi—were all at the University of Rome. In this connection, Carol Parikh [1991, 17] wrote in the third chapter of his biography of Oscar Zariski (1899–1986) that "in the fall of 1921 the University of Rome was the most important center of algebraic geometry in the world." Other mathematical subjects were also well represented at the University of Rome: Gaetano Scorza (1876–1939) was professor of algebra, Mauro Picone (1885–1977) and Leonida Tonelli (1885–1946) professors of analysis, and Enrico Bompiani (1889–1975) professor of differential geometry. As the historian Reinhard Siegmund-Schultze found in documentary sources, during the second half of the 1920s Wickliffe Rose and George David Birkhoff wrote some reports for the *International Education Board* in which Rome was identified as the third center of European mathematics, after Paris and Göttingen.[19]

A large number of foreign students came to Rome to do research, which they could do thanks to the close links of Levi-Civita and Volterra with the *International Education Board*. Some of these students were Griffith C. Evans (1887–1973, USA), who studied with Volterra; Pavel S. Alexandrov (1896–1982, USSR), with Enriques and Severi; Alexander Weinstein (1897–1979, USSR), with Levi-Civita; Oscar Zariski (1899–1986, USSR), with Castelnuovo and Enriques; Marcel Brelot (1903–1987, France), with Volterra; Paul Dubreil (1904–1994, France), with Enriques and Severi; Michel Guérard des Lauriers (1898–1988, France), with Levi-Civita; Szolem Mandelbrojt (1899–1983, France), with Enriques and Severi; Robert Mazet (1903–1991, France), with Levi-Civita; Joseph Pérès (1890–1962, France), with Volterra; André Weil (1906–1998, France), with Enriques and Severi; Octav Onicescu (1892–1983, Romania), graduated at the University of Rome in 1920; Gheorghe Vranceanu (1900–1979, Romania), with Levi-Civita (see [Tazzioli, 2001]); Dirk Struik (1894–2000, Netherlands), with Levi-Civita; Herbert Busemann (b. 1905), Werner Fenchel (1905–1988), Harald Geppert (1902–1945), Eric Kähler (1906–2000), and Hans Lewy (1904–1988) (Germany), all with Levi-Civita; Vaclav Hlavaty (1894–1969, Czechoslovakia), with Levi-Civita; and Aurel Friedrich Wintner (1903–1958, Hungary), with Levi-Civita.[20]

For many of them Rome had a particular charm, as Struik described in a letter to Levi-Civita (February 14, 1926):

> La vie à Göttingen est aussi monotone que la vie à Rome était pleine de distractions. Pour l'étude cela n'est pas un mal, mais on ne veut pas toujours étudier. Il y a ici plusieurs mathématiciens étrangers, dont plusieurs sont aussi étudiants Rockefeller, parmi ces mathématiciens est M. Solberg, que vous souvenez peut-être du congrès de Delft et qui s'occupe du problème des courants tourbolents. Parmi les étrangers qui sont passés par Göttingen j'annonce M. Serge Bernstein de Charkov, qui exprima, à nous, son grand contentement de ce que vous lui aviez toujours envoyé vos tirages à part, même dans le temps du plus grand isolement de la Russie.

Hans Lewy, another Rockefeller student and assistant of Richard Courant (1888–1972), wrote to Levi-Civita on December 18, 1931—even in the names of Busemann and Fenchel:

[19] See [Siegmund-Schultze, 2001, pp. 37, 39, 44, 268, and passim].

[20] Levi-Civita's testimonial on Wintner's research is reported in Appendix D.

Dear Professor,

We often remember our stay in Rome, we speak about our Roman friends and our inspirations there. And we always realize that the charm that we connect to the idea of Rome comes—in our thoughts—from the affable hospitality of your home: we always could count on your interest and quick help. In this particular moment we want to express to you our sincere feelings of gratitude towards you and your family.[21]

The "particular moment" concerns the oath to Fascism that professors were obliged to swear, an official dictate that led to the disintegration of the Roman group of mathematicians. Francesco Severi (1879–1961) discussed the idea of the *new oath* with the philosopher Giovanni Gentile (1879–1944), the "great old" of Italian culture during Fascism. According to Severi, the oath would put on the same level *new* and *old* Fascists, and former anti-Fascists as well; thanks to the new oath, Severi hoped that all professors would become equal—"real professors" and not "half professors," who were excluded from any boards for appointing new professors (*Commissioni di concorso*).

Such ideas are expressed in a letter written by Severi to Gentile on February 2, 1929.[22] In the same period, Severi was climbing to the top of his scientific and political career—in a short time he was to replace Federigo Enriques (1871–1946) as a member of the Academy of Italy and took over leadership of Italian mathematics thanks to Gentile's support.

The "trick" of Severi and Gentile was intended to solve the question of "anti-Fascist intellectuals," but did not take into account the *Fascistization* of the Universities. The first victim of the new oath was Vito Volterra, whose opposition to Fascism is well known and documented. In 1931, only a few university professors—among them Volterra—refused to swear fidelity to the new government and, in consequence, were forbidden to teach. In contrast, Levi-Civita signed the oath to Fascism in spite of his personal convictions. Familiar matters (career and family) and worry about his "school" prevailed over moral considerations.

Probably Severi was in good faith when he wrote his letters to Gentile about the oath to Fascism. However, he was very mistaken. The oath was the first step toward a total Fascistization of social, political and cultural Italian life. In 1935 the "Unione Matematica Italiana"—founded by Volterra in 1922—expelled Castelnuovo, Leonida Tonelli (1885–1946), Giulio Vivanti (1859–1949), and Volterra himself from the Scientific Commission. Finally, in 1938, the Racial Laws were decreed; they had the effect that about a hundred professors were expelled from Italian universities. Some of them were well known in the academic world; we shall only mention Federigo Enriques, Tullio Levi-Civita, Guido Fubini, Beppo Levi (1873–1964), Alessandro Terracini (1889–1968), Beniamino Segre (1903–1977), Arturo Maroni (1878–1966), Guido Horn d'Arturo (b. 1879), and Eugenio Curiel (1912–1943) (see [Israel and Nastasi, 1998]).

Because of his resistance to the "lust of assent," after the oath Levi-Civita was no longer appointed to commissions of public competitions for professorships. It should be noted that in the years 1903–1928, he had been a member of such commissions in 11 public competitions for professorships and 8 commissions for advancements from extraordinary to ordinary professor. Even if after the Racial Laws Levi-Civita and

[21] Caro Professore,

spesso si ricorda tra noi il nostro soggiorno di Roma, si parla degli amici nostri Romani, delle ispirazioni ivi avute. E si finisce sempre col constatare che l'attrattiva che—nel nostro pensiero—si lega all'idea di Roma, proviene in modo particolare dall'affabile ospitalità della Sua casa: checché ci fosse, si poteva contare sul Suo interesse nonché sulla Sua prontezza di aiutarci. Appunto in questo momento ci importa di esprimerLe i nostri sinceri sentimenti di gratitudine verso di Lei e la Sua famiglia. (In [Nastasi and Tazzioli, 2000, 315–316].)

[22] The letter is published in [Guerraggio and Nastasi, 1993].

the old Roman group of mathematicians were *forgotten*—they were formally excluded from all kinds of commissions—they still had considerable influence on Italian mathematics and academic politics in general. In fact, Levi-Civita was co-director of *Annali di Matematica*, the most important journal of mathematics in Italy together with *Rendiconti del Circolo matematico di Palermo*; moreover, he was one of the main scientific leaders of the *Accademia dei Lincei* and the author of the renewal—in 1936—of the prestigious *Accademia Pontificia delle Scienze* (see [Nastasi, 2000]).

2.2. International appreciation

"He was invited abroad many times, in order to give lectures on the results of his own research," wrote Levi-Civita about himself with great modesty in 1938, when he tried to be excepted from the Racial Laws. He certainly deserved general and high esteem for his remarkable contributions to different research fields. Here, we only consider his specific role in the scientific organization of the "International Congress of Applied Mechanics," a role showing that Levi-Civita was one of the most important Italian mathematicians in the opinion of the international mathematical community.

To appreciate Levi-Civita's role in the origin of this congress,[23] it is important to note that *internationalism* had always been a *real* ideal in his life and work. For example, he disagreed with the decision to exclude Axis scientists from international meetings. There are some interesting letters between Arnold Sommerfeld (1868–1951) and Levi-Civita, dated just after the First World War. In a letter (Munich, November 27, 1920) to Levi-Civita, Sommerfeld complained about injustices suffered by German people during the war and after the peace. Levi-Civita responded on December 9, 1920[24]:

> Very dear Collegue,
>
> I thank you for your letter dated November 27th (which I received just today) and for your authoritative—and then flattering—interest on my research of relativity.
>
> I appreciate very much the sincerity of your political statements. I do not enter in detail and do not specify the points of my disagreement exactly, but I want to tell you with the same sincerity a concise profession of faith.
>
> I have always been, and not only in science, a convinced internationalist and, in consequence of this ideal, I consider au dessus de la melée all nationalisms indistinctly. I have the same opinion about the nationalisms preceding and following the horrible war, which upset Europe in so ruinous a way.
>
> I understand well that you do not share my opinion. But we agree on an essential point—and I am pleased about it—that scientific relationships and personal relationships between scientists coming from different countries should not be perturbed by contingences or memories of national or state disagreements.
>
> On this occasion I send you my most friendly sentiments and the expression of my very high esteem. (...)[25]

[23] The event is reported in [Battimelli, 1996].

[24] The letter by Levi-Civita to Arnold Sommerfeld is contained in the "Archives of Deutsches Museum."

[25] Chiarissimo Collega,

La ringrazio per la cordiale Sua lettera del 27 u.s. (pervenutami soltanto oggi) e per l'interessamento autorevole, e quindi tanto più lusinghiero, accordato alle mie ricerche di relatività.

Apprezzo altamente la franchezza delle Sue dichiarazioni di carattere politico; e, senza entrare nel merito e specificare in conformità i punti del mio dissenso, tengo a farle a mia volta con altrettanta franchezza una sintetica professione di fede. Io sono sempre stato, non soltanto in scienza, un internazionalista convinto, e, in base a tale idealità, considero au dessus de la melée tutti indistintamente i nazionalismi, nonché i precedenti e i conseguenti della orribile guerra che ha sconvolto l'Europa in modo così disastroso.

Vedo bene che Ella non si pone su questo terreno. Però in un punto essenziale—e me ne compiaccio vivissimamente—ci troviamo in pieno accordo: nel convincimento che i rapporti scientifici in genere e quelli personali tra gli studiosi d'ogni paese, e tra noi due in particolare, non debbano essere comunque turbati da contingenze o ricordi di divergenze nazionali o statali.

Mi è grata l'occasione per riaffermarle i più amichevoli sentimenti e l'espressione della mia altissima stima. (...)

Levi-Civita's reputation as an internationalist was one of the reasons that he was asked to support scientific cooperation in Europe after the First World War. Theodore von Kármán (1881–1963), the director of the Institute of Aerodynamics in Aachen and one of Levi-Civita's interlocutors, worked for a "German–Italian" enterprise, which was to involve also French and English scientists. In a letter to Levi-Civita (April 12, 1922), von Kármán suggested the organization of an international scientific meeting on problems of fluid dynamics, a subject with many applications to different fields—engineering, mathematics, and physics. He was aware of the difficulties of his enterprise and therefore proposed an *informal* meeting instead of an *official* Congress. Von Kármán intended to gather scientists supporting his idea and coming from Germany, Austria, and the neutral countries (for example, the Swedish Carl William Oseen); while Levi-Civita—in case he supported the initiative—would promote the enterprise in "Roman and English" countries.

Von Kármán asked Levi-Civita's help for political and scientific reasons. Levi-Civita's internationalism was indeed well known and von Kármán hoped that his initiative could really become *international* thanks to German–Italian cooperation. In addition, Levi-Civita was very interested in both theoretical and practical science; such an attitude—together with the choice of the subject "fluid dynamics"—would be instrumental in bringing together "theorists" and "technicists" in the first International Congress of Applied Mechanics. Von Kármán's point of view was well described in his slogan, "turning engineering design into engineering science."

In his answer (April 21, 1922), Levi-Civita enthusiastically accepted von Kármán's proposal and suggested that the meeting should be held "as a personal initiative of some scientists, very few in number and as international as possible qualitatively." He immediately asked many Italian colleagues and of course many of them declined his proposal. For example, Gaudenzio Fantoli (professor of Hydraulics at the Polytechnic in Milan) and Vito Volterra had both taken part in the foundation of the *International Research Council*, whose statute excluded scientists coming from the "Countries of the Entente" from new international institutions. Marcel Brillouin and Richard von Mises also refused to support the German–Italian enterprise. Nevertheless von Kármán and Levi-Civita persevered and in September 1922 about 30 scientists—the majority from Germany and Austria—met in Innsbruck. Italian scientists were the only "victors" of the First World War who participated; they were Levi-Civita's students and friends: Modesto Panetti, Giulio De Marchi, Bruto Caldonazzo, Umberto Cisotti, and of course Levi-Civita himself.

In Innsbruck it was decided to organize periodic meetings not only on fluid dynamics but in all other fields of applied mechanics—the "International Congresses of Applied Mechanics" were born. The first of them was in Delft (April 22–25, 1924) and the organizers were Burgers and Biezeno.

In the *Proceedings* of the Meetings held in Innsbruck and in Delft, Levi-Civita published the text of his lectures: "Über die Transportgeschwindigkeit in einer stationären Wellenbewegung" (Innsbruck, 1922) [Levi-Civita, 1924] and "La détermination rigoureuse des ondes permanentes d'ampleur finie" (Delft, 1924) [Levi-Civita, 1925a], which indicate Levi-Civita's resumption of interest in fluid dynamics. In the same period, he published the Note "Risoluzione dell'equazione funzionale che caratterizza le onde periodiche in un canale molto profondo" [Levi-Civita, 1922b], where the theorem of existence and uniqueness for irrotational waves was proved. All these works were systematically organized in a work [Levi-Civita, 1925b] published in *Mathematische Annalen* and later completed by M.L. Dubreil-Jacotin, the wife of Paul Dubreil (see Section 1.2).

In 1922 Levi-Civita was awarded the Sylvester Medal by the Royal Society of London, a prize never given to any foreign mathematician before him, and in 1930 he was elected a foreign member of the

Royal Society. In 1931, he was the only Italian member of the board of the *Zentralblatt für Mathematik*, the most important journal of mathematical references at the time. (In 1938, after the promulgation of the Racial Laws, Levi-Civita was expelled from the journal, and Severi and Enrico Bompiani took his place; as a consequence, some of the most important members of the journal—among them the director Otto Neugebauer (1899–1990)—resigned, and another review journal was founded, the *Mathematical Reviews*.)

In 1932 and 1934 the well-known "Hadamard Seminar" devoted its meetings to the work of Levi-Civita and his students on "adiabatic invariants." In 1933 Levi-Civita delivered his first series of lectures in the United States, as a guest of Brown University, of the American Mathematical Society, of the American Association for the Advancement of Science (and of the Committee of Chicago for the celebration of "A Century of Progress Exposition"), and of Princeton University. Levi-Civita was invited by R.C. Archibald, C.R. Adams, M.H. Ingraham, W.C. Graustein, and L.P. Eisenhart.

In 1935, Levi-Civita was a guest at various Soviet scientific institutes (Institute of Tensorial Calculus, Institute of Aerodynamics, Institute of Theoretical Astronomy, Academy of Sciences, and Mathematical Society). The ambassador of the fascist government reported to the Italian government that Levi-Civita's lectures in Moscow (from May 13 to June 2) and in Kiev (June 5–12) were greatly appreciated.[26]

Levi-Civita returned to the United States in 1936 for the third centenary of Harvard University and lectured also in Princeton and at the Rice Institute. During his period in the USA Levi-Civita ran the risk of causing an incident with the fascist government. In fact, when he was in Houston giving some lectures at Rice, he granted the "Houston Chronicle" an interview where the superiority of the American university system with respect to the Italian system was strongly asserted. The Italian consul asked Levi-Civita to send a clarification to the newspaper and Levi-Civita obeyed. Nevertheless, this clarification was so hesitant that it seemed to strengthen—and not to correct—his former claim. The consul then asked the Italian government to take strict measures against Levi-Civita. But the Director of the Ministry of National Education suggested that it was not advisable to react too strongly, since Levi-Civita was a scientist "of world-wide renown." It was better to recall him moderately "in a formal or in a semi-official way."[27]

In the same year, the 11th International Mathematics Congress met in Oslo. But the Fascist government prevented Italian scientists from going to Norway, since it was a "*sanctionist* country!" In spite of his forced absence, Levi-Civita was appointed a member of the Commission which was to award the two subsequent Fields medals—the first ones were awarded on the occasion of the 1936 Congress to Lars Ahlfors (1907–1996) of Harvard University and Jesse Douglas (1897–1965) of MIT.

In 1937 Levi-Civita was in Lima, where he gave scientific lectures. We know something of his journey from the informative note of the Italian ambassador,[28] and from some notes and newspaper cuttings kept in Levi-Civita's Archives. Levi-Civita arrived in Lima on August 4 and delivered nine lectures on relativity and two lectures on the trigonometry of curvilinear triangles on a surface at the University of

[26] The note of the Italian Ambassador in Moscow (June 6, 1935) is in Levi-Civita's *dossier*, kept in the "Historical Archives of the University of Rome." The document also reports that Levi-Civita gave four lectures in Kiev, where he was received very cordially—the Academy of Sciences of Kiev held a dinner party in honor of him, and the *VOKS* (the Society for Cultural Relations with Foreign Countries) held a party in the most important hotel in the town.

[27] See the report of Giuseppe Giustini, general Director of the Ministry of National Education, dated March 1937, which is kept in Levi-Civita's *dossier* (Historical Archives of the University of Rome).

[28] The note (on September 20, 1937) is contained in Levi-Civita's *dossier* (Historical Archives of the University of Rome).

San Marco. In Lima he "was heartily well liked by everybody" and the dean of the Faculty of Sciences gave "a party in the Country Club of Lima [in his honor], where the most important people from the cultural and intellectual milieu were invited." Nevertheless, the Italian ambassador—the marquis Talamo Atenolfi—noticed that Levi-Civita had dissimulated his aversion to Italian government by entrenching himself behind his "unilateral technicality." In fact, the ambassador wrote in the conclusion of his note:

> Permit me to notice that the above-mentioned university professor carried on in a laudable way during the time he spent in Lima. Nevertheless, for our propaganda in this country it would be better to put in evidence elements which are not strictly technical and more suitable for spreading our thought in countries of low culture—such as this country is.[29]

In September 1938 the Racial Laws were promulgated just as the Fifth International Congress of Applied Mechanics was meeting in the USA. Levi-Civita was prevented from participating in the meeting and his absence was deplored by his colleagues, as is shown in some letters of Struik and Unsaker.

In 1938 again, Levi-Civita was invited by the Swedish Academy of Science to nominate a candidate for the Nobel Prize of Physics for the year 1939. Since he was informed that Fermi would get the Nobel Prize for the year 1938, Levi-Civita proposed Max Born (1882–1979). Born was to win the Nobel Prize later, in 1954, for his fundamental research on statistical mechanics, in particular for his statistical interpretation of the wave function. Levi-Civita's motivation was in total accordance with sentiments he had expressed in a letter to Carl Wilhelm Oseen (1879–1944) on December 12, 1938[30]:

> Le Comité Nobel pour la Physique m'a fait l'honneur de m'inviter à proposer un candidat pour le prix de physique de 1939. En Octobre dernier, lorsque j'ai reçu une telle invitation, j'ai pensé à M. Fermi, et j'avais même recueilli tout ce qu'il faut pour appuyer dûment cette proposition. L'heureuse attribution à M. Fermi du prix pour 1938 fait naturellement tourner mon attention vers d'autres savants.
>
> Je signale M. Born, actuellement professeur à l'Université d'Edinburgh, dont j'admire la géniale activité, qui est largement et fondamentalement répandue dans les domaines les plus fécondes de la physique mathématique moderne: relativité; dynamique des réseaux cristallins et structures des corps solides; fondements algébriques de la mécanique quantique; interprétation probabilistique des fonctions d'ondes de Broglie; sans négliger les ouvrages didactiques et d'haute vulgarisation. Ce qui me paraît surtout mériter le prix Nobel est l'éclatante, et en même temps si simple et naturelle, explication de l'électron. On sait que les équations linéaires de Maxwell n'admettent pas l'électron, c'est-à-dire une charge électrique qui ne soit pas condamnée à éclater. Ce péché originel a été corrigé par M. Born. [...] L'importance et la fécondité de cette conception se manifestent non seulement dans la définitive mise au point de la théorie classique, mais aussi dans le progrès remarquable de son développement a permis de réaliser en fournissant la clef pour la quantisation de l'électromagnétisme.
>
> A cause de la campagne antisémitique, qui sévit ici, je n'ai plus assez de contacts avec le monde académique italien pour me renseigner si complètement que le demande votre lettre du Septembre dernier. Je pourrais le faire en m'adressant à l'auteur lui-même; mais l'initiative est délicate; et je préfère vous informer d'avance, étant naturellement sous-entendu que votre silence ne m'encouragera pas à donner suite à la démarche.

2.3. Concluding remarks

Levi-Civita wrote in a letter (May 4, 1939) to one of his students, Gheorghe Vranceanu, who was to become one of the founders of the Romanian school of differential geometry:

[29] Mi permetto di far presente che per quanto il predetto cattedratico abbia svolto una attività encomiabile durante il tempo passato a Lima, sarebbe più opportuno, ai fini della nostra propaganda in questo Paese, far venire degli elementi meno strettamente tecnici e più adatti per la diffusione del nostro pensiero in paesi, come questo, a media culturale bassa.

[30] The letter of Levi-Civita to Oseen is contained in the Archives of the Swedish Royal Academy of Science in Stockholm.

> I live as a retired person and I do not move: except in summer, however, if my general conditions allow me to move. As you maybe know, Jews have been completely expelled from the Italian cultural life; in particular, I will not participate in the "Volta Congress" and will not be in Rome in September.[31]

These sentences show the sorrow of a person, who had devoted all his life to science and now was prevented from entering the Library of the Mathematic Institute where he had worked for over 20 years, and who depended on a few colleagues to read a volume of *Annali di Matematica pura e applicata*, the journal of mathematics where he had been on the editorial staff for decades.

As a consequence of the racial laws, the Italian mathematical community disintegrated. The following professors were forbidden to teach[32]:

- Guido Ascoli, full professor of *mathematical analysis*, University of Milan;
- Ettore Del Vecchio, professor of *general and financial mathematics*, University of Trieste;
- Federigo Enriques, full professor of *higher geometry*, University of Rome;
- Gino Fano, full professor of *analytical geometry*, University of Turin;
- Guido Fubini Ghiron, full professor of *analysis*, Polytechnic of Turin;
- Guido Horn d'Arturo, full professor of *astronomy*, University of Bologna;
- Beppo Levi, full professor of *mathematical analysis*, University of Bologna;
- Tullio Levi-Civita, full professor of *rational mechanics*, University of Rome;
- Arturo Maroni, full professor of *analytical geometry*, University of Pavia;
- Giorgio Mortara, full professor of *statistics*, University of Milan;
- Beniamino Segre, full professor of *analytical geometry*, University of Bologna;
- Alessandro Terracini, full professor of *analytical geometry*, University of Turin.

According to Law Number 1390 on September 5, 1938, concerning the so-called *Legislative measures in defence of the race in the Fascist school*, all of them were excluded from the Italian school system and from academies and cultural institutions. Moreover, about 10% of the members (27 members exactly) were excluded from the *Italian Mathematical Society* (*UMI*).

The effects of the Racial Laws were more than catastrophic for Italian science as a whole. As a consequence of the Fascist decrees of 1938, the Italian school of physics was wiped out. Indeed, the following physicists were forced into exile: Bruno Rossi, Enrico Fermi (who was awarded the Nobel Prize), Emilio Segré, Sergio De Benedetti, Ugo Fano (the son of the mathematician Gino), Eugenio Fubini (the son of the mathematician Guido), Leo Pincherle (the grandson of the mathematician Salvatore, who founded functional analysis in Italy), Bruno Pontecorvo, and Giulio Racah. Franco Rasetti, the "oldest" physicist, and together with Fermi one of the "Panisperna street boys" (who were highly reputed in the world for their research in nuclear physics) was "Aryan," but he did not want to remain in Italy after the Racial Laws and emigrated to Canada.

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[31] Io faccio il pensionato, e sto fermo: non però nell'estate, sempreché le condizioni generali permettano una qualche mobilità. Come forse Lei sa, gli ebrei sono stati estromessi da qualsiasi partecipazione alla vita culturale italiana: in particolare quindi, non parteciperò al Convegno Volta, né sarò a Roma in Settembre. (In [Nastasi and Tazzioli, 1999, 363–364].)

[32] These all were university professors. In addition, the following *liberi docenti* were also expelled from Italian universities: Alberto Mario Bedarida (algebraic analysis in Genoa), Giulio Bemporad (astronomy in Turin), Bonaparte Colombo (infinitesimal analysis in Turin), and Bruno Tedeschi (financial and actuarial mathematics in Trieste).

The school of biology founded by Giuseppe Levi in Turin was wiped out. Besides Levi himself, two of his students—who were later to be awarded the Nobel Prize—departed: Salvatore (then Salvador Edward) Luria, and Rita Levi Montalcini. In addition to them, we mention another "Aryan" winner of the Nobel Prize, Renato Dulbecco, who also belonged to Levi's school of biology. Further, 18 full professors and 119 lecturers of medicine were expelled from the Italian universities. Chemistry lost two of the principal animators of industrial chemistry, Giorgio Renato Levi and Mario Giacomo Levi.

The international image of Italian mathematical research suffered much in consequence of the racial laws. To compensate for this, the Fascist government donated a large amount of money—about 50.000 Italian lire—for the purpose of publishing the collected papers of the great Italian mathematicians of the Risorgimento and instituting the National Institute for Higher Mathematics in Rome (law approved on July 13, 1939).

But that did not suffice. The replacement of Levi-Civita on the editorial staff of the *Zentralblatt* caused a severe international reaction.[33] In a letter to F. Springer dated December 5, 1938, Oswald Veblen (1880–1960) pointed out the gravity of the situation—international scientific solidarity was strongly affected, the mathematical world was openly divided into two parts (Italy and Germany on one side, and the other countries on the other side), and the *Zentralblatt* could not be considered "a useful scientific enterprise" any more.[34] The project of abstracting and reviewing mathematical literature had to move to the USA, not for nationalistic reasons, but because the "required freedom from political influence" was guaranteed only in that country "at present time and probably for a considerable period in the future."

In the last years of his life, in spite of his moral and physical depression, Levi-Civita remained faithful to the ideal of scientific internationalism and helped colleagues and students who were victims of anti-Semitism; thanks to him, many of them found positions in South America or in the USA. In many letters addressed to him, his help was requested; from the documents of his *Archive* it clearly emerges that he intervened in favour of Leo Finzi (letter from London on September 11, 1938); Guido Fubini—who obtained a position at the Institute for Advanced Study in Princeton (see Appendix C); Alessandro Terracini—who obtained a position in Tucuman, Argentina; Berud Steinlerger (letter from Zurich on October 2, 1938); and Enrico Volterra (1909–1973)—the son of Vito, who was the assistant of Levi-Civita from 1933 to 1938, and after many difficulties secured a position at the University of Rosario, Santa Fè, Argentina.

Levi-Civita's death on December 30, 1941, was ignored by the Italian mathematical community. He was one of the most eminent professors in Italy for over 40 years and attracted students coming from all countries, whom he encouraged with patience and nobility. Kindness and modesty were manifestations of his soul. Many people benefited from his kindness and retained an ineffaceable memory of his extraordinary personality. Evidence of this side of Levi-Civita's personality—which is quite unknown—is found in the letters and documents contained in his *Archive*. We produce some of them in Appendixes A–D.

[33] The event is reported in detail in [Siegmund-Schultze, 1994].

[34] Veblen's letter, together with Neugebauer's letters to Levi-Civita and to Springer, are published in [Nastasi, 1991].

Acknowledgments

We thank Umberto Bottazzini for his suggestions and advice, which allowed us to improve the first version of the paper. We also thank Jens Høyrup and Craig Fraser for checking our English.

Appendix A. Letter of reference for Dirk Jan Struik probably addressed to Jan Arnoldus Schouten

[Rome], le 8 Février 1932

Très éminent et cher Collègue,

Puisque j'écris plus couramment le français que l'allemand, vous voudrez sans doute me permettre de répondre en français à votre lettre du 5 de ce mois.

C'est avec plaisir que je vais exprimer mon avis au sujet des travaux hydrodynamiques de M. Struik. J'apprécie également l'activité géométrique de cet auteur, mais il serait évidemment oisif d'en parler, en s'adressant à un maître de la matière, à bon droit universellement reconnu.

Je me bornerai à rappeler que, ayant eu fréquemment des entretiens scientifiques avec M. Struik, à l'époque où il était à Rome comme boursiste Rockefeller, j'ai pu me rendre compte non seulement de son talent, de sa pénétration analytique et mécanique, et de ses connaissances très étendues dans plusieurs branches des mathématiques, mais aussi de sa forte capacité de travail, et de ses belles qualités humaines.

Sa contribution principale à l'hydrodynamique est la résolution rigoureuse (voir notamment la mémoire des Math. Ann., B 95, 1926, pp. 595–634) du problème des ondes permanentes, irrotationnelles (c'est-à-dire sans tourbillons) dans un canal de profondeur quelconque. On avait auparavant établi l'existence d'ondes du type susdit dans l'hypothèse idéale d'une profondeur infinie. Il fallait surmonter des sérieuses difficultés de concept et de calcul pour adapter au cas réel où la profondeur est finie, ou même petite, la méthode qui avait conduit au but dans le cas limite d'une profondeur infinie. M. Struik y est parvenu d'une manière heureuse et bien nette. Il a d'abord découvert une ingénieuse démonstration d'existence, fournissant en le même temps un algorithme constructif d'approximations successives. Ensuite il a imaginé un second algorithme pratiquement plus avantageux, et il en a tiré plusieurs conséquences, rigoureuses, ou approchées, très intéressantes. Elles comprennent, peut-on dire, l'étude complète, même au point de vue quantitatif, de l'influence de la profondeur du canal sur la propagation des ondes périodiques, notamment sur la forme du profil, sur les vitesses moyennes au fond et à la surface, sur le transport global, etc. Je n'hésite pas à affirmer qu'il s'agit d'une recherche tout à fait remarquable.

Quant aux autres jeunes chercheurs hollandais, auxquels vous faites allusion dans votre lettre, je n'en puis absolument rien dire.

Veuillez agréer, mon cher Ami, les sentiments les plus cordiaux de ma considération distinguée.

Appendix B. Letter of reference for Alexander Weinstein

Rome le 2 Juillet 1934

M. le Professeur Gibson, F.R.S.
Academic Assistance Council—Londres

M.A. Weinstein a travaillé jadis, pendant plusieurs mois, à Rome témoignant ses beaux talents et sa large pénétration, qui l'ont conduit, avant, alors et après, a apporter des contributions tout à fait remarquables à des questions fondamentales d'analyse et d'hydrodynamique. Je me bornerai à signaler, à titre d'exemple, deux recherches, qui ne sont pas les plus importantes dans la production de M. Weinstein, mais que j'apprécie particulièrement puisque elles se rapportent à un domaine où j'ai travaillé moi-même. Il s'agit d'une étude sur l'onde solitaire en seconde approximation, mentionnée déjà par M. Lamb ("Hydrodynamics," p. 425 de la 6.ème édition); et de sa récente mise au point, avec de très intéressants compléments, du problème du sillage provoqué par un obstacle circulaire.

M. Weinstein, qui aurait mérité depuis longtemps une chaire universitaire, est une victime, d'abord des difficultés de l'après-guerre et ensuite du racisme naziste.

Je sais que deux éminents savants parisiens, M. Hadamard et M. Villat, ont très chaleureusement appuyé la demande adressée par M. Weinstein pour qu'il lui soit accordé une bourse de Votre Comité, qui a, à tant d'égards, si hautement bien mérité de la science et de l'humanité. Cette bourse, s'ajoutant à quelque profit occasionnel dans l'Institut pour la mécanique des fluides, lui permettrait de continuer à travailler à Paris, dans l'attente (qui est à espérer assez courte) d'une situation stable et satisfaisante en Suisse ou ailleurs.

Appendix C. Some letters about the search for a position for Guido Fubini in the United States following the proclamation of the Racial Laws

[Levi-Civita to Oswald Veblen in Princeton]

Roma, 30.10.1938

Dear Professor Veblen,

As you probably know, all Italian teachers, racially Jew, from elementary schools to Universities, have been dismissed; furthermore some other antisemitic rules have been established, or foreseen in this country.

Among the Italian mathematicians thus stroken some one's, as Enriques and I, have attained the retiring age; but there are some distinguished colleagues still young, or hardly mature, who had expected a better future for them and their families.

This being premised as a matter of fact, I remember that once you have said to me that unfortunately there is no place more in America (better in the USA) for European mathematicians, except perhaps for men of international renown. Such is undoubtedly my dear friend Professor Guido Fubini, till now full professor of Calculus in the Polytechnicschool and lecturer for higher analysis at the University of Turin.

I scarcely need to remind his genial work in differential geometry, where he has open new pathways with a great deal of papers and a book on projective differential geometry; his researches on movements and on conformal and geodesic groups being equally fundamental. After Volterra, he is considered the most penetrating and ingenuous living Italian analyst, having supplied essential contributions in many vital fields: for instance, automorphic functions of several variables, also in connection to Hermitian forms; old and new problems in the calculus of variations; integral equations with non symmetric polar kernel (said also equations of the third kind); partial differential equations (extension of the methods of Riemann and Picard to equations of higher order; asymptotic behaviour of certain differential equations encountered in the radiotechnique; modern critics of foundations (multiple integration, mean-theorem, minimizing principle).

In the last time his chief interest turned himself toward the mathematical theory of elasticity and other questions of engineering science; he has found the way of attacking and resolving the difficult problem of the flexion of beams with curvilinear axis, and has otherwise in print (with professor Albenga) a big volume on engineering mathematics and its applications.

Professor Fubini is (as long as he will not be expelled as a Jew) a National Member of the Academies dei Lincei, of Turin, of the XL, and several others. He is a vivid man of profound intelligence and ready wit, who was celebrated among Italians scholars as a model of clear and brilliant expositor.

Of course, if it is possible to offer him a satisfactory, stable situation, nothing better; but it would be in any case an objectively desiderable policy for the Institute of Advanced Study to invite him as a temporary fellow. Professor Fubini, who is now in Paris (Albany Hotel, 202 Rue de Rivoli) would accept with great pleasure and gratitude such an invitation, not only as a conspicuous honour, but also as a precious occasion given to him of transferring to America his family (wife and two sons).

With highest compliments to Mrs. Veblen, I beg you receive my best thanks, greetings and excuses if I due call upon the scientific and human mission of the Institute, through your illuminate patronage.

Very faithfully yours (. . .)

[Abraham Flexner to Levi-Civita]

Princeton, December 1, 1938

Dear professor Levi-Civita:

I am enclosing a copy of a letter which on the recommendation of the mathematical group of the Institute I am sending to Professor Fubini. It occurs to me that Professor Fubini may no longer be in Paris or at the same address. If such should be the case, would you be good enough to see that this copy, which I am sending you, reaches him.

We think often of the pleasure which we have derived from the visit of you and your wife, and we are hoping that an opportunity may recur to repeat it in the not too distant future.

With warmest greetings from my wife and myself and all your colleagues here, believe me

Sincerely yours,
Abraham Flexner

[Copy of letter of Flexner to Fubini]

December 1, 1938

My dear Professor Fubini:

Upon the recommendation of the professors of the school of Mathematics of the Institute for Advanced Study I have pleasure in inviting you to be a member of the Institute in the second term of the academic year, 1938–1939. The second term opens January 17 and closes May 1, 1939, You will receive a stipend of $500.00.

Under separate cover I am sending you the latest Bulletin of the Institute.

Sincerely yours,
(signed) Abraham Flexner

[Rough copy of letter by Levi-Civita to Flexner]

December 12, 1938

Dear Doctor Flexner,

Two days ago I have received with very grateful feelings your kind communication about the invitation that the Institute, benevolently following a fervid suggestion of mine, has addressed to Professor Fubini. A letter of him, which has just reached me, assures that your invitation is duly arrived at his hands.

I am delighted for the friendly recollections of my wife and myself, which you mention in so kind manner. We, in turn, remember well your exquisite courtesy and are glad to take the opportunity for sending to Mrs Flexner and you our best season wishes.

Sincereley yours

Appendix D. Letter of reference for Aurel Friedrich Wintner

Rome, March 7, 1939

I am very glad to have the opportunity of expressing the highest appreciation for the acute, ingenious, very penetrating work already done by Professor Aurel Wintner and my firm belief in his capacity to make further important contributions in the field of probability and of fruitful applications illustrated in his plan for work.

I have been in contact with Dr. Wintner here in Rome, where he has spent one year with a Rockefeller fellowship and so have had the possibility of recognizing his excellent qualities of character as well as his quiet, fertile, thorough and exceptionally productive work. He had already written at that time a very good book on matrices and resolved attractive questions of real importance in celestial mechanics and his beautiful activity has afterward still improved.

Very happy has been for him and for theoretical astronomy his visit to Copenhagen, where he succeeded in giving mathematical proofs of very fundamental and striking phenomena detected by the danish school through numerical computations.

In recent times he has continued and improved these researches approaching also the more recent and delicate theories centered on the ergodic theorem. Just in this field lie the applications he proposes to accomplish in collaboration with Prof. Wiener.

I conclude recommending without reserve Prof. Wintner for the fellowship for which he applies.

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